EKONOMSKI FAKULTET RIJEKA

Ivana Filipovića 4 • 51000 Rijeka
tel +385 51 355 111
fax + 385 51 212 268
http://www.efri.uniri.hr
e-mail efri@efri.hr

PRIKAZ REZULTATA ISTRAŽIVANJA

Ime i prezime doktoranda:	**Maria Čuljak, univ.spec.oec.**
Matični broj:	**231/19**
Smjer:	**Doktorski studij ekonomije i poslovne ekonomije**
Upisana godina:	2023/2024

Tema doktorske disertacije

Analiza i otpornost procjenitelja distribucije prinosa, volatilnosti i kovolatilnosti dioničkih tržišta pomoću visokofrekventnih podataka
Analysis and robustness of estimators of the distribution of returns, volatility and covolatility of stock markets using high frequency data

Mentor:	Prof.dr.sc. Saša Žiković
Komentor:	Prof.dr.sc. Josip Arnerić

Područje istraživanja doktorske disertacije (opis i razrada rezultata istraživanja temeljem prihvaćene prijave teme doktorske disertacije te povezanost rezultata istraživanja u dokazivanju postavljenih hipoteza; posebno objasniti način na koji su uvaženi i razriješeni prijedlozi, mišljenja i kritike Povjerenstva koje je ocijenilo prijavu teme doktorske disertacije): **Quantitative Finance**

1. **Detaljan osvrt na komentare, prijedloge i pitanja sa obrane prijave teme doktorske disertacije**

The defense of application of dissertation topic was on March 1st 2022.
Regarding the questions from doc.dr.sc. Zaninović:
"In the first part/paper of the dissertation you compare different option pricing models (Shimko's, mixed lognormal d., Edgeworth's), i.e. their predictive powers. What is the actual contribution of your approach to the existing field of knowledge? Application of the models on high-frequency data?
Given that your data comes from financial markets of developed countries, can you argue that the same methodology (for example, for integrated variance and covariance estimation) can be applied to financial markets of Central and Eastern European countries?"
The scientific contribution in application of option pricing models on high-frequency trading data because it allows for the estimation of intraday volatility, which can be used as an input to option pricing models. For instance, one then can calculate realized volatility from high-frequency price data and use it to update option prices more frequently throughout the trading day. This research contributes to the existing literature in two ways: i) finding the benchmark of the "true" density function using high-frequency data within Kernel estimator and ii) determining the predictive accuracy of the option pricing models, which is the purpose of this research. The comparison of benchmark density function against estimated risk neutral probability functions produces applicative results for market participants and public authorities, respectively. Also, it provides better insights into high-frequency data issues. The same methodology proposed (in this dissertation) for integrated variance and covariance can

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be used on data of financial markets of Central and Eastern European countries. Those markets are less liquid so some adjustment should be made. One must handle any data issues, such as missing observations, outliers, or non-trading hours, and also it may be needed to interpolate data points if there are long gaps between trades. For estimating realized variance (which is a key component of the integrated variance), in less liquid markets it is just a matter of adjusting the sampling frequency.

Regarding the questions from Izv.prof.dr.sc. Ivana Tomas Žiković:

"U prijavi se navodi da će se koristiti procjenitelj s dvostrukom skalom kako bi se prevladao mikrostrukturni šum kad nema cjenovnih skokova. U nastavku se navodi da procjenitelj s dvostrukom skalom postaje pristran kada dođe do cjenovnih skokova. Nadalje se navodi da u prisutnosi mikrostrukturnog šuma i cjenovnih skokova, procjenitelj s dvostrukom skalom ostaje nepristran. Molim detaljnije objašnjenje navedenog odnosno svojstva navedenog procjenitelja.

Na temelju kojih kriterija su odabrana promatrana financijska tržišta CAC, AEX, MIB i DAX. Očekujete li da će Vaš model vrednovanja biti primjenjiv i za financijska tržišta koja nisu obuhvaćena uzorkom, odnosno hoćete li testirati model i na tržišta koja nisu trenutno obuhvaćena analizom (npr. zemlje centralne i jugoistočne Europe)?"

The robust two times scaled estimator has several elements of properties. One of the primary properties of the robust two times scaled estimator is its robustness. It is less sensitive to outliers and influential observations compared to traditional sample covariance estimators. This is achieved by using robust weighting schemes or ignore extreme observations when computing the covariance. The estimator is efficient, meaning it provides good estimates of the covariance matrix for multivariate data that may not follow a normal distribution and it also incorporates a bias correction term, which helps to reduce the bias in the covariance estimates. In the presence of jumps there will also be bias in practical applications of estimators that are not jump robust.

The robust two times scaled estimator is price jumps and microstructural noise robust. European financial markets CAC, AEX, MIB and DAX are examined and incorporated into the research of the dissertation because of high liquidity, economic significance, diversification and high quality data. The option pricing models used in this research are also applicable to other financial markets (for an example of Central and Eastern European countries). Due to less liquidity in those markets the key would be in preparation of data and dealing with sparsity by interpolation. Continuing my research I have not applied the used option pricing models to those markets.

Regarding questions from Izv.prof.dr.sc. Anita Čeh Časni:

"Why did you choose the Kernel estimate of the probability density function as an ex-post benchmark function with which to compare the implied ex-ante functions for different maturity dates of European options?

Which tests do you plan to use to empirically prove the presence of microstructural noise and price jumps in high-frequency dana?

What criteria do you use to determine the optimal sampling frequency?"

Using a kernel density estimate (KDE) of the probability density function (PDF) as an ex-post benchmark function can be valuable in various applications, particularly in the field of statistics and data analysis which is the case for this dissertation. KDE is a non-parametric method, meaning it does not assume a specific form for the underlying distribution. This flexibility is advantageous when the true distribution is not known. Also, the bandwidth parameter in KDE controls the smoothness of the estimate. This allows for a trade-off between

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bias and variance, and careful selection of bandwidth can result in a more accurate representation of the underlying distribution.

In this dissertation we were not proving the presence of microstructural noise, but suitable test would be the Hurst exponent that characterizes the "long-term memory" of a time series. A value of the Hurst exponent greater than 0.5 suggests a tendency for persistence in the data, while a value less than 0.5 suggests anti-persistence. Microstructural noise may lead to a lack of persistent patterns, resulting in a Hurst exponent close to 0.5. The test for price jumps we have included them through implementation of jump-diffusion models that explicitly incorporate jumps in the price process. These models can be estimated, and statistical tests can be conducted to check for the significance of the jump component.

By minimizing the Root mean squared error (RMSE) against the number of subsamples, the optimal sampling frequency (optimal slow time scale) is obtained.

Regarding the questions from Prof.dr.sc. Marija Kaštelan Mrak:

"Vezano uz znanstveni doprinos, smatrate li da će neka od vaših opažanja (zaključaka) biti moguće interpretirati i kroz opću mikroekonomsku teoriju tržišta i cijene? Naime, možda je moguće nešto zaključiti o formiranju cijena temeljem trenutnog odnosa snaga ili raspoloženja na tržištu?

Hoće li biti moguće temeljem podataka koje namjeravate prikupiti nešto pretpostaviti o ishodima primjene preporučene metode procjene kretanja cijena na razvoj financijskih tržišta; primjere na distribuciju prinosa i rizika među pojedinim kategorijama sudionika na tržištu odnosno njihovu spremnost na trgovanje u budućnosti? Kojoj su kategoriji izdavatelja ili ulagača tehnike određivanja cijena i procjene rizika koje predlažete mogle značajnije poboljšati vlastite sposobnosti odlučivanja? Na čiju „štetu"? Kakva su vaša očekivanja u pogledu odraza na sigurnost ulaganja (hrvatskih) obveznih mirovinskih fondova?"

In this dissertation we have not considered application of these models to microeconomic theory of market and prices. That is something that can be included in the future research. Application of the results of the data used in this research can make recommendations of methods in assessing price movements on other financial markets (not just the ones assessed in this dissertation). The distribution of returns and risks among certain categories of market participants or theirs willingness to trade in the future are shown through recommendations for which estimators to use for each observed market. Also, recommendations are given in third research paper for cryptocurrency market as well in showing how to construct the most profitable portfolio through sectoral categorization. The investments of (Croatian) mandatory pension funds are safe as they tend to invest purely in government bonds and most liquid stocks which are considered safer cluster of assets and do not have high volatility thus risk appetite.

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2. Sadržaj doktorske disertacije po poglavljima i potpoglavljima

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3. Prikaz rezultata istraživanja (preporuka obujma: od 16 do 48 stranica)

1. Introduction

During the last two decades we have been witnesses to a series of rolling crises starting from dot-com bubble, 2008 global financial crisis, COVID crisis, energy crisis, European sovereign debt crisis, Russo-Ukrainian War and Chinese property sector crisis. A key feature of these crises was a strong increase in cross-market and cross-assets correlations on regional or global scales. This characteristic is becoming an omnipresent danger signaling a growing effect of global financial contagion in international markets. The clear presence of regional/global financial contagion has not only practical implication on daily portfolio allocation and hedging strategies but also erodes the theoretical pillars of financial management insofar as the premise of diversifying away the risk by creating internationally diversified portfolios buckles at instances when we witness simultaneous drops across multiple asset classes on a global scale. During crisis periods, all market participants tend to seek refuge in currencies considered as safe havens. They convert their local cash holdings into these currencies to preserve value. The perceived safety of these currencies leads to an increase in their value, even if the events in question may not have had a substantial impact on that specific currency. Besides the U.S. Dollar, Swiss Franc and Japanese Yen which are well established in the literature as safe haven currencies i.e. a reliable stores of value during uncertain economic times cryptocurrencies also have the potential to act as safe haven asset due to their decentralized nature and lack of regulation. They have been discussed as a potential safe haven asset, especially in a recent years when their popularity spiked. Cryptocurrencies are not tied to any particular nation or organization, which makes them less vulnerable to government action or geopolitical threats. This research examines and concludes on which potential safe haven currency is a better safe haven asset over the observation period based on the defined benchmark Robust two times scaled estimator of covolatility. The methodology for determining the target relies on covolatility. In order to conclude on this objective we split our research in four parts. This doctoral dissertation deals with the field of financial econometrics, mainly with the analysis of time series of high-frequency data of financial assets. High-frequency data, observed at very short intervals of time, for example every minute or every second, provide more complete information not only about price movements, but also about trading activities, which allows a better understanding of phenomena in financial markets, such as the distribution of returns, volatility and covolatility. The assessment of precisely these phenomena using high-frequency data is the way we conclude on the main objective of this dissertation. We start by examining distribution of returns and finding a data driven benchmark of the "true" density function for major market indices in consideration. Following that, we investigate whether the Robust two times scaled estimator is superior among alternative estimators of volatility by utilizing high-frequency data on a 1 second time scale over a 7-year period. Next we examine potential safe haven asset, cryptocurrencies market by comparing sectoral cryptocurrency portfolios with the benchmark CRIX index. Finally, we determine the best covolatility estimator and identify the best performing safe haven currencies compared to general market movements.

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2. Predictive accuracy of option pricing models considering high-frequency data.

In the first part, high-frequency data are used to estimate the probability density functions for the selected maturity dates of the European put and call options with the aim of comparing them with the implied density functions, derived ex-ante (before the maturity date). The objective of this research is to employ high-frequency data in determining the forecasting power of option pricing models. High-frequency data are used here to provide a reference probability density function that would be a benchmark for comparison purpose. Besides of high-frequency data observed every minute, the put and call options data on the stock market indices CAC (Cotation Assistée en Continu), AEX (Amsterdam Exchange index), MIB (Milano Indice di Borsa), and DAX (Deutscher Aktien index) are considered in this study. All of these data were obtained from the Thomson Reuters financial service. The research is conducted in two phases. The first phase includes estimating the implied probability density functions at the expiration date using options data. The second phase deals with comparing the estimated probability density functions against the reference density function based on high-frequency data, obtained by Kernel estimation method. Employing the Kernel density estimation method on observed high-frequency data in real time, provides an applicative contribution and thus a great advantage over other studies which mostly rely on simulation data. The used models are from a class of non-parametric, parametric and semi-parametric option pricing models: Shimko model, Mixture Log-Normal model and Edgeworth expansion model, respectively. The main objective of the research is to evaluate their predictive accuracy and to select the most appropriate one, not only that best fits the data, but has the highest predictive accuracy simultaneously. Prior to the analysis, the raw data is cleaned as follows. It is assumed that, at each exercise price, call and put options are available in pairs. Cleaning is done by taking a sample that satisfies more criteria than required by the bid price to be greater than zero. Therefore, there is usually a big difference between the available call and put options prices and the actually used. If there are less than ten exercise prices for which we have call and put options, then the probability density function will not be estimated by any of the models used in this paper. The observed stock indices are CAC, AEX, MIB and DAX, i.e. the French, Dutch, Italian and German market index, respectively. Financial instruments used are call and put options on the major indices of the listed financial markets on combinations of options trading dates and options expiration dates in 2018 (Table 1).

Table 1 Options trading dates and expiration dates with respect to four stock market indices

Year 2018	Options expiration dates		
Options trading dates	July 20	August 17	September 21
March 23			AEX, DAX, MIB
April 20			AEX, CAC
May 18	DAX		AEX, CAC, DAX, MIB
June 22	AEX, CAC, DAX,	AEX, DAX	AEX, DAX

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fax + 385 51 212 268
http://www.efri.uniri.hr
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	MIB		

Source: Thomson Reuters.

From Table 1 it can be noticed that options data are not available for all market indices at given trading dates. For example, on trading date May 18, 2018 options data are available for all four indices AEX, CAC, DAX and MIB with expiration on September 21, 2018 (maturity horizon approximately 4 months), but only for DAX index with expiration on July 20, 2018 (maturity horizon one month). Further, we estimate the implied probability density functions on the expiration dates of the Shimko model, Mixture Log-Normal model, and Edgeworth expansion. Additionally to options data, high-frequency data were obtained from the Thomas Reuters database for the same expiration dates, for which kernel density method is used to estimate the "true" probability density functions. Furthermore we provide results of comparison between estimated probability density functions, obtained by the three option pricing models, and benchmarks of the "true" probability density functions, obtained by Kernel estimation using high-frequency data. A graphical and analytical comparison is presented at each maturity date. The study that is most similar to our research compares three parametric density functions obtained by a mixture of two log-normal (MLN), Black-Scholes-Merton (BSM) and generalized beta (GB2) according to Arnerić et al. (2015). Mean square error (MSE) and absolute relative error (ARE) were used for pairwise comparison purpose only, neglecting the "true" probability density function that can be observed ex-post. Diebold – Mariano test (DM) is used to test which model has a lower MSE (Diebold et al., 1998). Abovementioned parametric models are usually overfitted making a wrong impression how these models fit the data. Due to unique characteristics of the proposed models we consider them to be sensitive to different maturity dates. Because semi-parametric and non-parametric approaches do not explicitly form the risk-neutral probability density function and there is no assumption about the function itself, this paper focuses on Shimko model (SM), Mixture Log-Normal model (MLN) and Edgeworth expansion model (EE). In our paper we implement out-of-sample comparison methods and determine predictive accuracy. Two tests were used here, the Diebold-Mariano test and the Kolmogorov-Smirnov test (Pauše, 1993). The results are provided for all combinations of selected trading and expiration dates. For the prices of call and put options the midpoints between bid and ask prices are taken. EURIBOR is taken as a risk-free interest rate, depending on the forecast horizon. The forecast horizon varies from 1 month to 6 months (Table 1). It was assumed that there were no dividend payments. Data processing was done in the "R Studio". The results for the AEX, CAC, DAX and MIB index are presented graphically from Figure 1 to Figure 18 (see Appendix). The figures present graphical comparison of implied probability density functions using three different option pricing models (MLN, EE, SM) and Kernel estimated probability density function based on high-frequency data, i.e. the "true" density (TD). From these figures it can be observed that Kernel density estimation is accurate enough to be used as a benchmark for comparative purpose out-of-sample and that mostly Shimko model fits the "true" density the best. This result is of great interest for investment industry so that analysts know which option pricing model to use when assessing the market expectations. Table 2 presents the comparison results obtained by two-sided Kolomogorov-Smirnov test and the Diebold- Mariano test, respectively.

Table 2 Comparison results from the Kolmogorov-Smirnov test and Diebold-Maraino test

Index / Expiration date / Trading date	Kolmogorov-Smirnov test			Diebold-Maraino test		
	TD-MLN	TD-SM	TD-EE	TD-MLN	TD-SM	TD-EE
AEX						

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fax + 385 51 212 268
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	August 17, 2018	0,26	0,54	0,67	-8,85	-7,51	-6,85	
	June 22, 2018	(p<0,05)	(p<0,05)	(p<0,05)	(p<0,05)	(p<0,05)	(p<0,05)	
	July 20, 2018	0,41	0,52	0,59	2,23	0,85	-2,21	
	June 22, 2018	(p<0,05)	(p<0,05)	(p<0,05)	(p<0,05)	(p>0,05)	(p<0,05)	
	September 21, 2018	0,38	0,61	0,72	-6,75	-7,81	-1,32	
	March 23, 2018	(p<0,05)	(p<0,05)	(p<0,05)	(p<0,05)	(p<0,05)	(p<0,05)	
	September 21, 2018	0,42	0,65	0,75	-5,74	-7,34	-4,24	
	April 20, 2108	(p<0,05)	(p<0,05)	(p<0,05)	(p<0,05)	(p<0,05)	(p<0,05)	
	September 21, 2018	0,37	0,59	0,65	-7,24	-8,21	-6,99	
	May 18, 2018	(p<0,05)	(p<0,05)	(p<0,05)	(p<0,05)	(p<0,05)	(p<0,05)	
	September 21, 2018	0,36	0,51	0,6	-5,41	-7,96	-7,38	
	June 22, 2018	(p<0,05)	(p<0,05)	(p<0,05)	(p<0,05)	(p<0,05)	(p<0,05)	
	CAC							
	July 20, 2018	0,53	0,80	0,38	-4,06	-7,55	-0,87	
	June 22, 2018	(p<0,05)	(p<0,05)	(p<0,05)	(p<0,05)	(p<0,05)	(p<0,05)	
	September 21, 2018	0,54	0,47	0,54	-7,04	-8,62	-5,41	
	April 20, 2018	(p<0,05)	(p<0,05)	(p<0,05)	(p<0,05)	(p<0,05)	(p<0,05)	
	September 21, 2018	0,31	0,35	0,53	-7,99	-9,82	-8,58	
	May 18, 2018	(p<0,05)	(p<0,05)	(p<0,05)	(p<0,05)	(p<0,05)	(p<0,05)	
	August 17, 2018	0,35	0,47	0,53	-18,06	-10,94	-7,00	
	June 22, 2018	(p<0,05)	(p<0,05)	(p<0,05)	(p<0,05)	(p<0,05)	(p<0,05)	
	DAX							
	July 20, 2018	0,37	0,55	0,38	-5,85	-1,45	-5,68	
	May 18, 2018	(p<0,05)	(p<0,05)	(p<0,05)	(p<0,05)	(p>0,05)	(p<0,05)	
	July 20, 2018	0,31	0,46	0,49	1,94	1,34	-0,76	
	June 22, 2018	(p<0,05)	(p<0,05)	(p<0,05)	(p>0,05)	(p>0,05)	(p<0,05)	
	September 21, 2018	0,34	0,15	0,13	0,34	-5,18	-7,70	
	March 23, 2018	(p<0,05)	(p<0,05)	(p>0,05)	(p>0,05)	(p<0,05)	(p<0,05)	
	September 21, 2018	0,30	0,31	0,35	6,45	3,10	-2,64	
	May 18, 2018	(p<0,05)	(p<0,05)	(p<0,05)	(p<0,05)	(p<0,05)	(p<0,05)	
	September 21, 2018	0,36	0,35	0,43	-4,35	-3,02	-2,64	
	June 22, 2018	(p<0,05)	(p<0,05)	(p<0,05)	(p<0,05)	(p<0,05)	(p<0,05)	
	MIB							
	July 20, 2018	0,22	0,31	0,29	1,35	-9,73	-6,34	
	June 22, 2018	(p<0,05)	(p<0,05)	(p<0,05)	(p>0,05)	(p<0,05)	(p<0,05)	

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tel +385 51 355 111
fax + 385 51 212 268
http://www.efri.uniri.hr
e-mail efri@efri.hr

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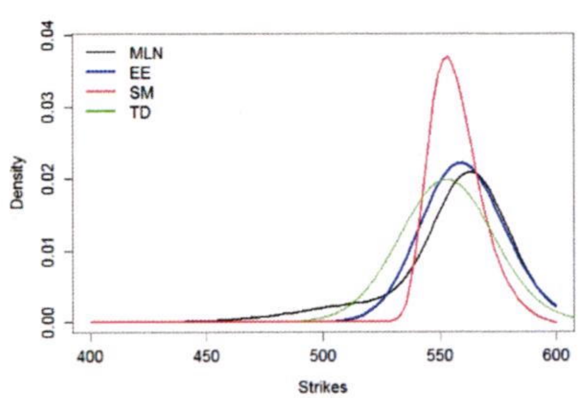
September 21, 2018 March 23, 2018	0,34 (p<0,05)	0,27 (p<0,05)	0,20 (p<0,05)	-2,50 (p<0,05)	-6,54 (p<0,05)	-6,91 (p<0,05)
September 21, 2018 May 18, 2018	0,38 (p<0,05)	0,35 (p<0,05)	0,28 (p<0,05)	-6,56 (p<0,05)	-7,84 (p<0,05)	-3,65 (p<0,05)

Source: Authors calculation using R Studio and Thomson Reuters data.

In most of the cases we reject the null hypothesis of Kolmogorov-Smirnov test that the estimated probability densities originate from the "true" density function. The null hypothesis is rejected at a significance level of 5% in the most cases except from DAX index on trading date of March 23, 2018 and maturity date of September 21, 2018. In that case we did not reject the null hypothesis of KS test at a significance level of 5% (p>0,05). This means that probability density function implied by the Shimko model and the "true" density function obtained by the Kernel estimator are the same.
Table 2 also provides aggregate DM test results for all observed stock indices and combinations of maturity and trading dates. DM is used to test the null hypothesis for the observed pricing models having the same forecasting ability. In the example of AEX stock index on maturity date August 17, 2018 and trading date June 22, 2018 the null hypothesis at a significance level of 5% was rejected. In an equal number of cases, the Mixture Log-Normal model, the Shimko model, and Edgeworth expansion model have been shown to have the same prognostic accuracy i.e. we did not reject the null hypothesis at a significance level of 5%. It is important to emphasize that in the case of DAX market index on the trading date of June 22, 2018 and expiration date July 20, 2018 all the models had the same prognostic accuracy. From the perspective that Kernel estimator provides referential probability density function, it can be concluded that Shimko model is the best fitting model out-of-sample when compared against the "true" density. Moreover, the null hypothesis of Kolmogorvo-Smirnov test was rejected in the most cases for all market indices and all combinations of trading and expiration dates. The results of the Diebold – Mariano test did not reject the null hypothesis implying that the models have the same predictive accuracy. According to the graphical presentations and the Kolomogorov-Smirnov test, we can conclude that the Shimko model predicts most accurately.

Figure 1 Comparison of risk-neutral densities obtained on June 22, 2018 for AEX index towards the true density of AEX index with a maturity date of August 17, 2018



Source: Authors calculation using R Studio and Thomson Reuters data.

Figure 2 Comparison of risk-neutral densities obtained at June 22, 2018 for AEX index towards the true density of AEX index with a maturity date of July 20, 2018

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tel +385 51 355 111
fax + 385 51 212 268
http://www.efri.uniri.hr
e-mail efri@efri.hr

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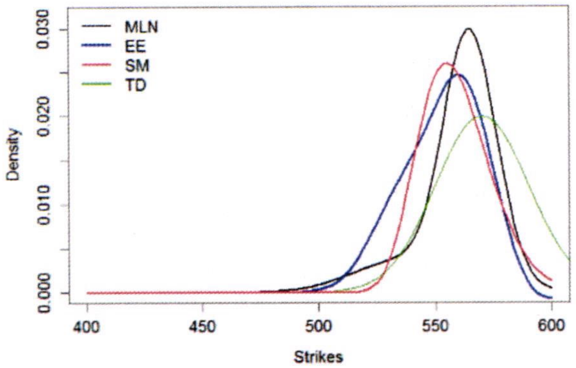
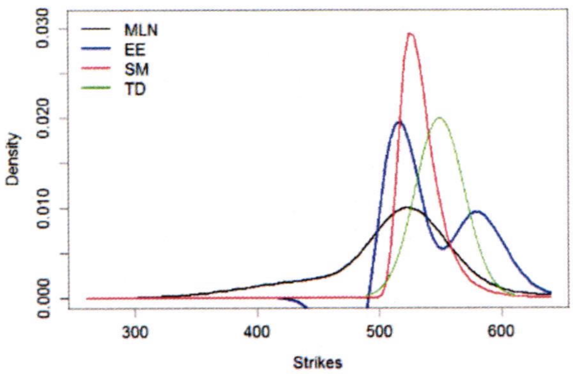


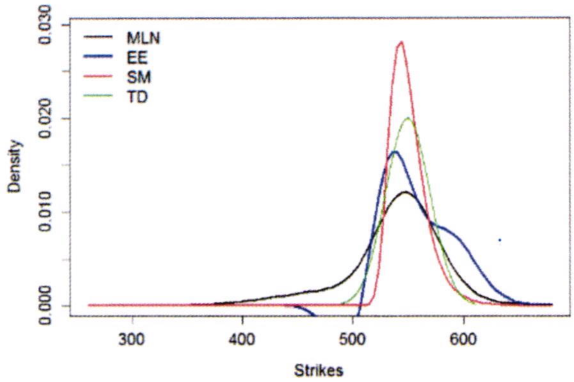
Source: Authors calculation using R Studio and Thomson Reuters data.

Figure 3 Comparison of risk-neutral densities obtained on March 23, 2018 for AEX index towards the true density of AEX index with a maturity date of September 21, 2018



Source: Authors calculation using R Studio and Thomson Reuters data.

Figure 4 Comparison of risk-neutral densities obtained on April 20, 2018 for AEX index towards the true density of AEX index with a maturity date of September 21, 2018



Source: Authors calculation using R Studio and Thomson Reuters data.

SVEUČILIŠTE U RIJECI
EKONOMSKI FAKULTET

Ivana Filipovića 4 • 51000 Rijeka
tel +385 51 355 111
fax + 385 51 212 268
http://www.efri.uniri.hr
e-mail efri@efri.hr

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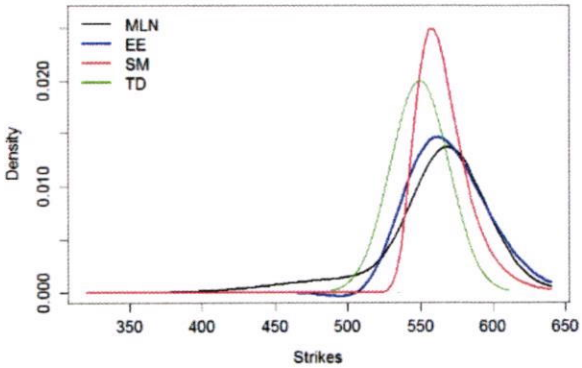
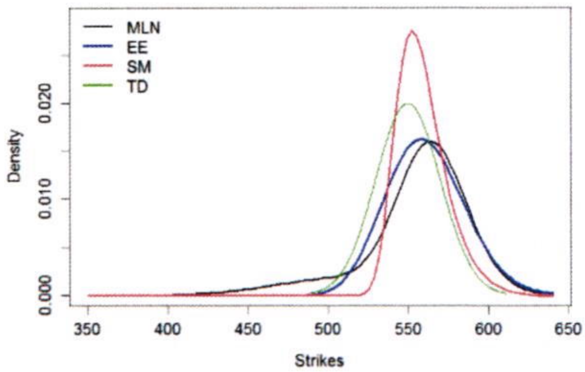
Figure 5 Comparison of risk-neutral densities obtained on May 18, 2018 for AEX index towards the true density of AEX index with a maturity date of September 21, 2018



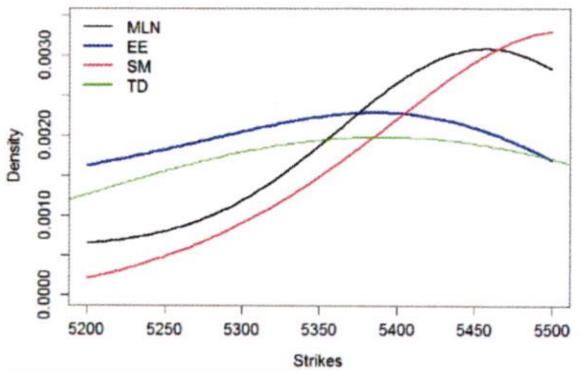
Source: Authors calculation using R Studio and Thomson Reuters data.

Figure 6 Comparison of risk-neutral densities obtained on June 22, 2018 for AEX index towards the true density of AEX index with a maturity date of September 21, 2018



Source: Authors calculation using R Studio and Thomson Reuters data.

Figure 7 Comparison of risk-neutral densities obtained on June 22, 2018 for CAC index towards the true density of CAC index with a maturity date of July 20, 2018



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EKONOMSKI FAKULTET

Ivana Filipovića 4 • 51000 Rijeka
tel +385 51 355 111
fax + 385 51 212 268
http://www.efri.uniri.hr
e-mail efri@efri.hr

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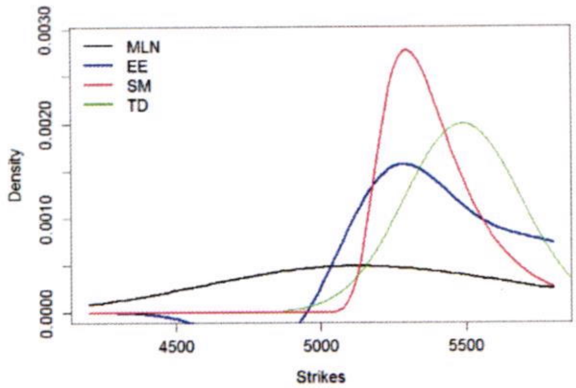
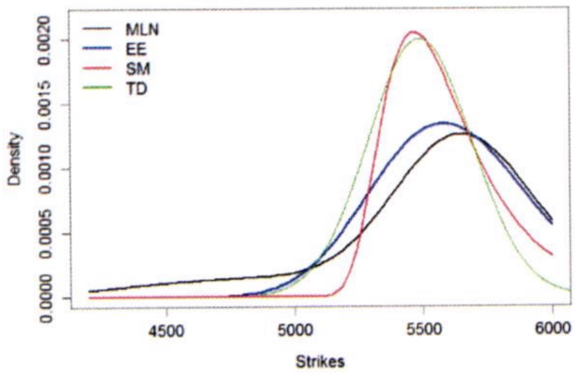
Source: Authors calculation using R Studio and Thomson Reuters data.

Figure 8 Comparison of risk-neutral densities obtained on April 20, 2018 for CAC index towards the true density of CAC index with a maturity date of September 21, 2018



Source: Authors calculation using R Studio and Thomson Reuters data.

Figure 9 Comparison of risk-neutral densities obtained on May 18, 2018 for CAC index towards the true density of CAC index with a maturity date of September 21, 2018



Source: Authors calculation using R Studio and Thomson Reuters data.

Figure 10 Comparison of risk-neutral densities obtained on June 22, 2018 for DAX index towards the true density of DAX index with a maturity date of August 17, 2018

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EKONOMSKI FAKULTET

Ivana Filipovića 4 • 51000 Rijeka
tel +385 51 355 111
fax + 385 51 212 268
http://www.efri.uniri.hr
e-mail efri@efri.hr

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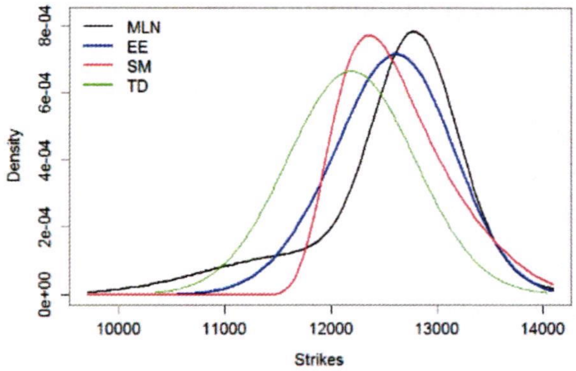
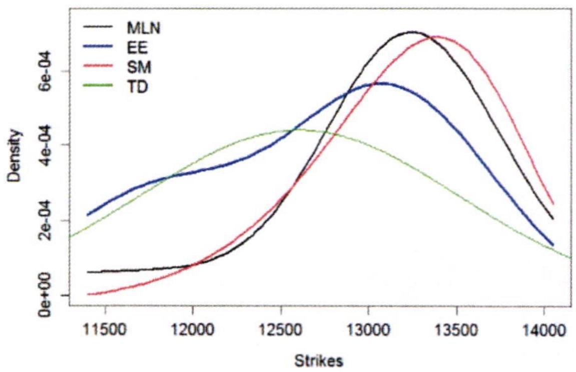


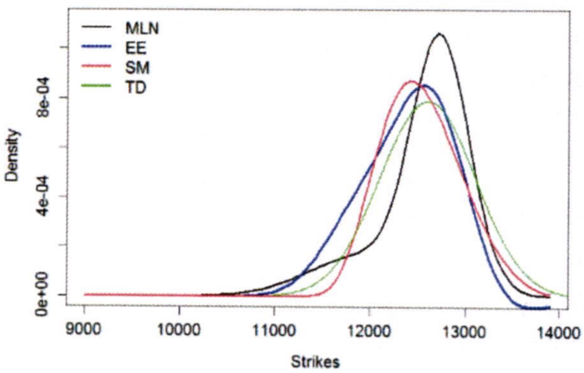
Source: Authors calculation using R Studio and Thomson Reuters data.

Figure 11 Comparison of risk-neutral densities obtained on May 18, 2018 for DAX index towards the true density of DAX index with a maturity date of July 20, 2018



Source: Authors calculation using R Studio and Thomson Reuters data.

Figure 12 Comparison of risk-neutral densities obtained on June 22, 2018 for DAX index towards the true density of DAX index with a maturity date of July 20, 2018



Source: Authors calculation using R Studio and Thomson Reuters data.

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EKONOMSKI FAKULTET

Ivana Filipovića 4 • 51000 Rijeka
tel +385 51 355 111
fax + 385 51 212 268
http://www.efri.uniri.hr
e-mail efri@efri.hr

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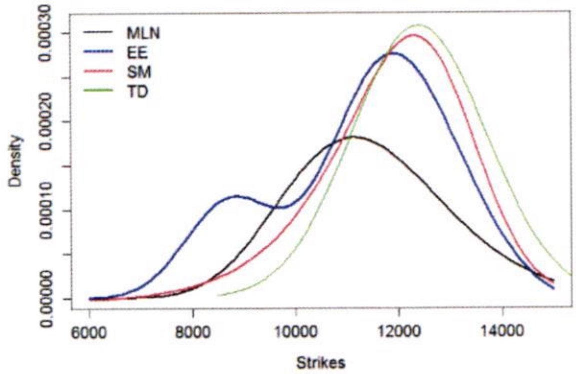
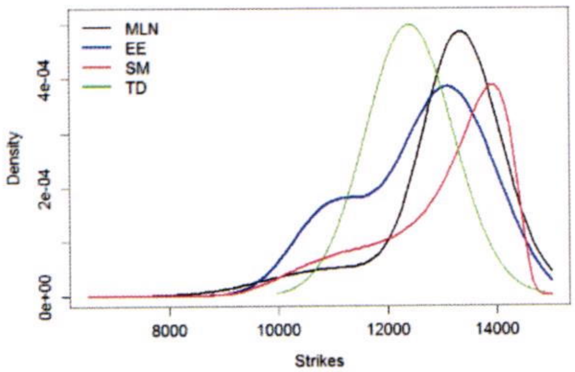
Figure 13 Comparison of risk-neutral densities obtained on March 23, 2018 for DAX index towards the true density of DAX index with a maturity date of September 21, 2018



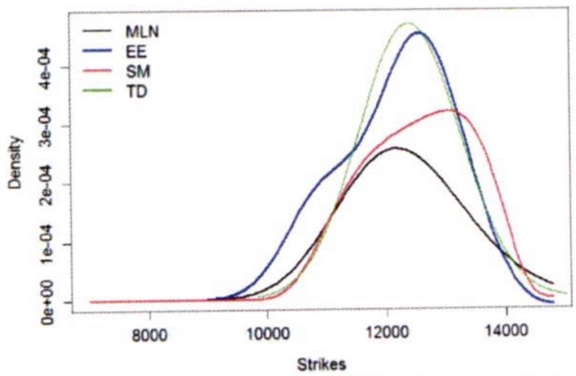
Source: Authors calculation using R Studio and Thomson Reuters data.

Figure 14 Comparison of risk-neutral densities obtained on May 18, 2018 for DAX index towards the true density of DAX index with a maturity date of September 21, 2018



Source: Authors calculation using R Studio and Thomson Reuters data.

Figure 15 Comparison of risk-neutral densities obtained on June 22, 2018 for DAX index towards the true density of DAX index with a maturity date of September 21, 2018



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EKONOMSKI FAKULTET

Ivana Filipovića 4 • 51000 Rijeka
tel +385 51 355 111
fax + 385 51 212 268
http://www.efri.uniri.hr
e-mail efri@efri.hr

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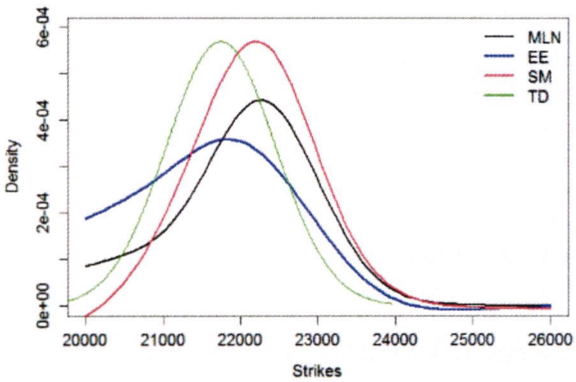
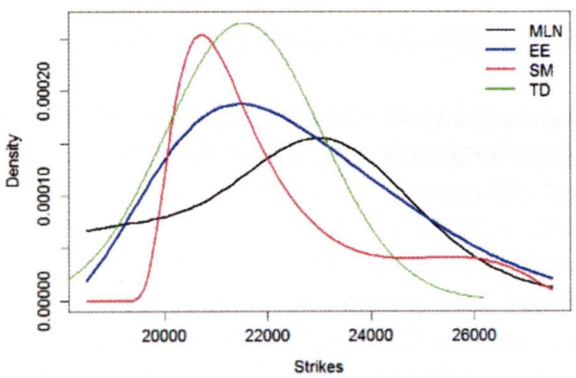
Source: Authors calculation using R Studio and Thomson Reuters data.

Figure 16 Comparison of risk-neutral densities obtained on June 22, 2018 for MIB index towards the true density of MIB index with a maturity date of July 20, 2018



Source: Authors calculation using R Studio and Thomson Reuters data.

Figure 17 Comparison of risk-neutral densities obtained on March 23, 2018 for MIB index towards the true density of MIB index with a maturity date of September 21, 2018



Source: Authors calculation using R Studio and Thomson Reuters data.

Figure 18 Comparison of risk-neutral densities obtained on May 18, 2018 for MIB index towards the true density of MIB index with a maturity date of September 21, 2018

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EKONOMSKI FAKULTET

Ivana Filipovića 4 • 51000 Rijeka
tel +385 51 355 111
fax + 385 51 212 268
http://www.efri.uniri.hr
e-mail efri@efri.hr

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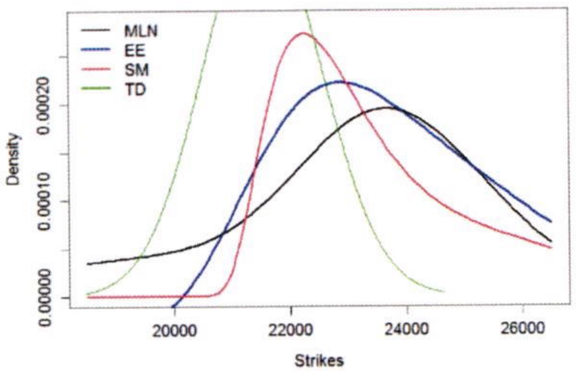

Source: Authors calculation using R Studio and Thomson Reuters data.

This research contributes to the existing literature in two ways: i) finding the benchmark of the "true" density function using high-frequency data within Kernel estimator and ii) determining the predictive accuracy of the option pricing models, which is the purpose of this research. The comparison of benchmark density function against estimated risk neutral probability functions produces applicative results for market participants and public authorities, respectively. Moreover, research cognitions are offer better insights into high-frequency data issues.

3. Is Jump Robust Two Times Scaled Estimator Superior among Realized Volatility Competitors?

In the second part of the dissertation, the integrated variance (true but unknown population variance) of the selected stock market indices is estimated using high-frequency data. It goes without saying that we are talking about the variance of the return of financial assets, that is, the standard deviation of the return, which is identified with the term volatility. We investigate whether the robust two times scaled estimator is superior among alternative estimators of integrated variance by utilizing high-frequency data on a 1 second time scale over a 7-year period. Two groups of estimators are considered, estimators which are robust to microstructure noise and estimators robust to price jumps. The performance of estimators is tested on stock market indices from developed European countries. The analyzed indices are liquid, heavily traded and exhibit intensive intraday activity. The optimal sampling frequency of each estimator is determined with respect to the tradeoff between its bias and the variance and individually adjusted to features of each stock market index. In addition to probability integral transformation test and Mincer-Zarnowitz regression, upper tail dependence from the Gumbel copula is considered as an appropriate pairwise comparison measure. The superiority of robust two times scaled estimator is proven for all the analyzed markets with respect to the optimal slow time scale sampling frequency. In this paper high-frequency data of developed stock markets is considered, covering the period from January 4, 2010 to April 28, 2017 for Germany, Italy, France and UK. The data was provided by Thomson Reuters Tick History. The fast time scale sampling frequency of 1 second is determined in advance, according to data availability of the observed financial markets within the shortest, nonempty and equidistant intervals. In order to define the optimal slow time scale sampling frequency, the root mean square error (RMSE) of the proposed estimator is used. The RMSE of each estimator was calculated as the sum of its squared bias and its variance, and afterwards the RMSE was minimized with respect to slow time scale frequency which corresponds to the number of

SVEUČILIŠTE U RIJECI
EKONOMSKI FAKULTET

Ivana Filipovića 4 • 51000 Rijeka
tel +385 51 355 111
fax + 385 51 212 268
http://www.efri.uniri.hr
e-mail efri@efri.hr

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subsamples.
The descriptive statistics of the data including the optimal sampling frequency are presented in Table 3.

Table 3 Description of high-frequency data from January 4, 2010 to April 28, 2017

European market	Stock index	No. of trading days	No. of 1 sec. observations	Optimal slow time scale
Italy	MIB	1862	42773317	10 sec.
Germany	DAX	1863	56650069	20 sec.
France	CAC	1878	4665980	13 sec.
UK	FTSE	1850	48920222	30 sec.

In Table 3 the number of trading days and number of 1-second observations are given. These numbers vary depending on the observed European market. The intraday data taken into consideration was during trading hours from 9 a.m. until 5:30 p.m. for all developed European markets.

Table 4 Realized volatility estimators

IV estimator	Formulation
Realized variance	$RV_t^{\Delta} = \sum_{i=1}^{n_t} r_{t_i}^2$
Bipower variation	$BPV_t^{\Delta} = \frac{\pi}{2} \sum_{i=2}^{n_t} \lvert r_{t_{i-1}} \rvert \lvert r_{t_i} \rvert$
Minimized block of two returns	$MinRV_t^{\Delta} = \frac{\pi}{\pi - 2} \frac{n_t}{n_t - 1} \sum_{i=2}^{n_t} min(\lvert r_{t_{i-1}} \rvert, \lvert r_{t_i} \rvert)^2$
Medianized block of three returns	$MedRV_t^{\Delta} = \frac{\pi}{6 - 4\sqrt{3} + \pi} \frac{n_t}{n_t - 2} \sum_{i=2}^{n_t - 1} median(\lvert r_{t_{i-1}} \rvert, \lvert r_{t_i} \rvert, \lvert r_{t_{i+1}} \rvert)^2$
Average subsampled realized variance	$ARV_t^{\Delta,k} = (\frac{n_t}{n_t - k + 1}) \frac{1}{k} \sum_{j=1}^{k} \sum_{i=1}^{n_t} r_{t_{ij}}^2$
Two times scaled realized variance	$TSRV_t^{\Delta,k} = (1 - \frac{n_t - k + 1}{n_t k})^{-1} (\frac{1}{k} \sum_{j=1}^{k} \sum_{i=1}^{n_t} r_{t_{ij}}^2 - \frac{n_t - k + 1}{n_t k} \sum_{i=1}^{n_t} r_{t_i}^2)$

SVEUČILIŠTE U RIJECI
EKONOMSKI FAKULTET

Ivana Filipovića 4 • 51000 Rijeka
tel +385 51 355 111
fax + 385 51 212 268
http://www.efri.uniri.hr
e-mail efri@efri.hr

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Robust two times scaled realized variance	$RTSRV_t^{\Delta,k,\theta} = (1 - \frac{n_t - k + 1}{n_t k})^{-1} c_\theta (\frac{1}{k} \sum_{j=1}^{k} \sum_{i=1}^{n_t} r_{t_{ij}}^2 I_i(\theta) - \frac{n_t - k + 1}{n_t k} \sum_{i=1}^{n_t} r_{t_i}^2 I_i(\theta))$
Hayashi-Yoshida realized variance	$HYRV_t^{\Delta} = \sum_{i=1}^{n_t} r_{t_i}^2 (I^i)$

Eight estimators of integrated variance (IV) are presented in Table 4.

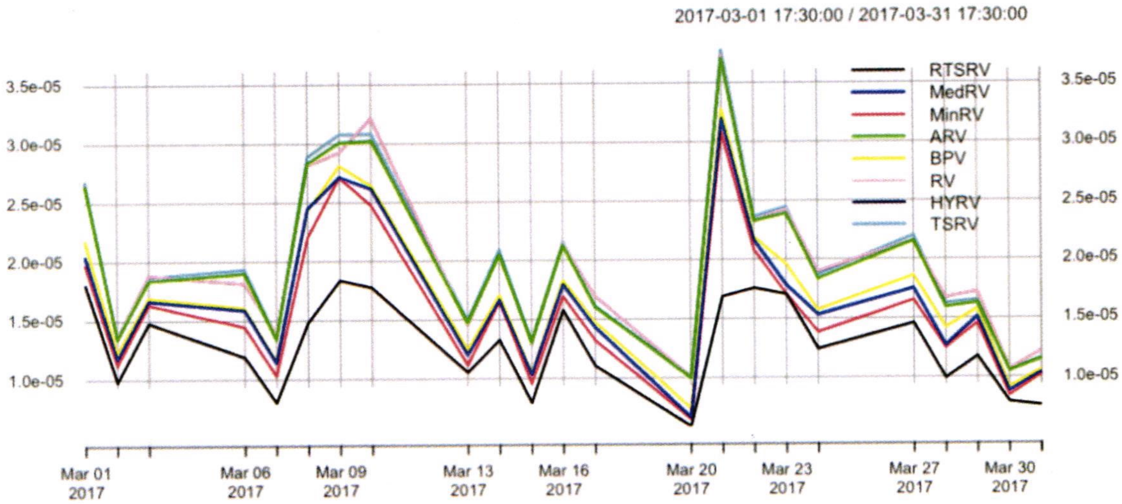

Figure 19 All realized volatility estimators for German stock market

Figure 19 presents eight realized volatility estimators for the German stock market index (DAX). For better visualization of the difference between estimators, the data frame used was from March 1, 2017 until March 31, 2017.

After the calculation of realized volatility estimators and $RTSRV_t^{\Delta,k,\theta}$, three comparison methods are conducted in order to determine which volatility estimator fits best with the benchmark. These three comparison methods are Mincer-Zarnowitz regression, probability integral transformation test and Gumbel copula upper tail dependence. Mincer-Zarnowitz regression is based on the overall performance of the realized volatility models. PIT test was performed as a density goodness of fit procedure in order to test how the examined realized volatility estimators preform in comparison to $RTSRV_t^{\Delta,k,\theta}$. The upper tail dependence was used because it examines what happens in the extreme values or tails.

The $RTSRV_t^{\Delta,k,\theta}$ is defined as a benchmark because it is confirmed that $RTSRV_t^{\Delta,k,\theta}$ is robust to market microstructure noise, jumps and non-synchronous trading in the intraday stock price series. This advantage from other realized volatility estimators was verified by the reduced bias and mean square error in a simulation study (Boudt & Zhang, 2013).

Each comparison method comprises fitting seven realized volatility estimators to the benchmark that

SVEUČILIŠTE U RIJECI
EKONOMSKI FAKULTET

Ivana Filipovića 4 • 51000 Rijeka
tel +385 51 355 111
fax + 385 51 212 268
http://www.efri.uniri.hr
e-mail efri@efri.hr

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is robust two times scaled estimator ($RTSRV_t^{\Delta,k,\theta}$). The optimal sampling frequency was selected for each European market, based on minimizing the root mean squared error (RMSE) (A¨ıt-Sahalia et al., 2005; Arnerić, Matković & Sorić, 2019). That is when the $RTSRV_t^{\Delta,k,\theta}$ becomes unbiased to microstructure noise. For the calculation of the RV, it is suggested to use returns that are sampled as often as possible because we get the maximum amount of information from data that is ultra-high-frequency. However, if the sampling frequency is as high as it can be, it leads to a bias problem due to microstructure noise (Oomen, 2005). There is a trade-off between the bias and efficiency while determining the sampling frequency. One must establish the optimal sampling frequency for an observed financial market in order to reduce the bias but for a volatility estimator to still remain efficient (Ait -Sahalia et al., 2005; Bandi & Russel, 2008). In the presence of jumps there will also be bias in practical applications of estimators that are not jump robust. So it will have an effect on sampling frequency which is also influenced by market structure, liquidity and microstructure noise. The way to increase the efficiency of estimators is to sub-sample (taking the average of an estimator across all possible sub-samples) (Ait -Sahalia et al., 2005; Andersen et al., 2012). With microstructure robust estimators the optimal sampling frequency is obtained by minimising the mean square error (MSE) (Arnerić et al., 2019; Zhang, 2011). As seen in Figure 20, the optimal sampling frequency was established for each European market. For Germany it is 20 seconds, for UK it is 30 seconds, for Italy it is 10 seconds and for France it is 13 seconds. It shows the root mean squared error (RMSE) of the robust two times scaled estimator $RTSRV_t^{\Delta,k,\theta}$ for each observed developed European market index against the number of subsamples. By minimising the RMSE against the number of subsamples, the optimal sampling frequency (optimal slow time scale) is obtained.

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EKONOMSKI FAKULTET

Ivana Filipovića 4 • 51000 Rijeka
tel +385 51 355 111
fax + 385 51 212 268
http://www.efri.uniri.hr
e-mail efri@efri.hr

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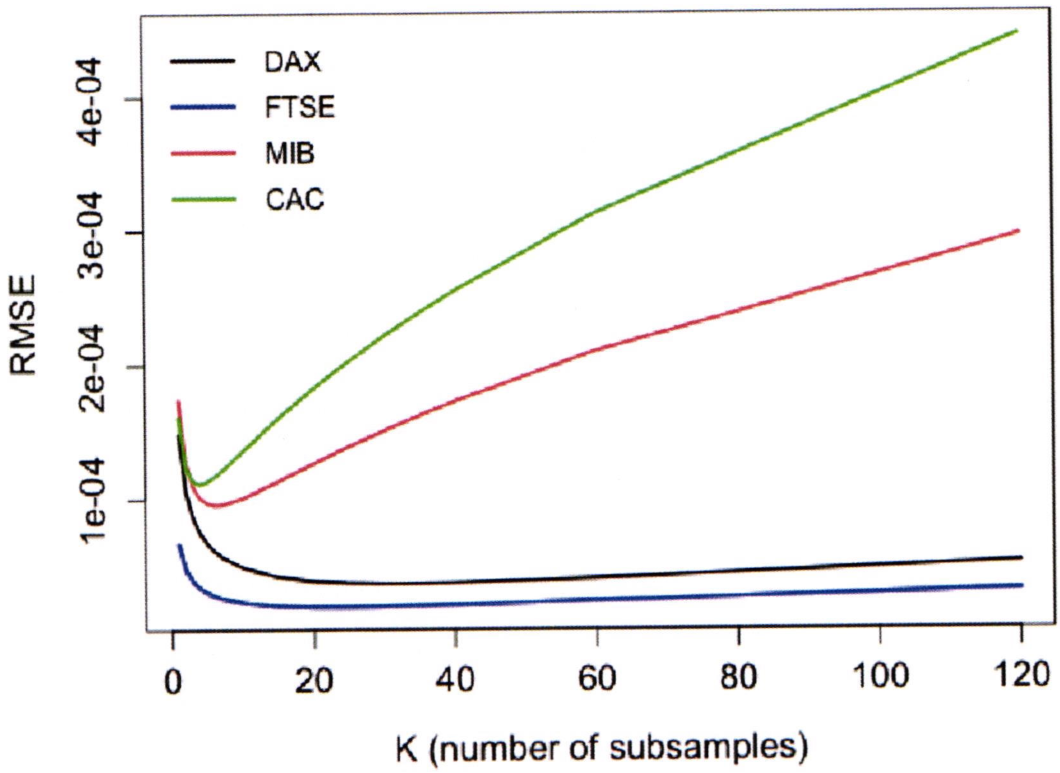

Figure 20 Root mean squared error of the RTSRV estimator for each European market index against the number of subsamples

The comparison methods used were Mincer-Zarnowitz regression, probability integral transformation (PIT) test and Gumbel copula upper tail dependence. The results of the first two methods for comparison of the realized volatility estimators indicate that RV_t^{Δ}, $TSRV_t^{\Delta,k}$, $ARV_t^{\Delta,k}$ and $HYRV_t^{\Delta}$ underestimate the performance of estimates during severe stress and price jumps. Therefore, the results do not give a clear answer which estimator fits the best to the benchmark. In that case, the upper tail dependence is a favorable method to use because it takes into account extreme values. Each competing volatility estimator was tested against $RTSRV_t^{\Delta,k,\theta}$ using Mincer Zarnowitz regression, PIT test and upper tail dependence measure within the Gumbel copula. Firstly, the Mincer-Zarnowitz regression was used where it was tested how well the volatility estimators fit to $RTSRV_t^{\Delta,k,\theta}$:

$\hat{\sigma}^2_{RTSRV} = \beta_0 + \beta_1 \hat{\sigma}^2_t + \epsilon_t$

More specifically, it was tested whether $\beta_0 = 0$ and $\beta_1 = 1$. The results, including chi-squared test statistic and p-value, are presented in Table 5 below. For four European markets and seven volatility estimators the null hypothesis was rejected at significance level of 5%. This showed how seven observed volatility estimators do not fit well to $RTSRV_t^{\Delta,k,\theta}$. Figures 21.a and 21.b are scatter plots

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EKONOMSKI FAKULTET

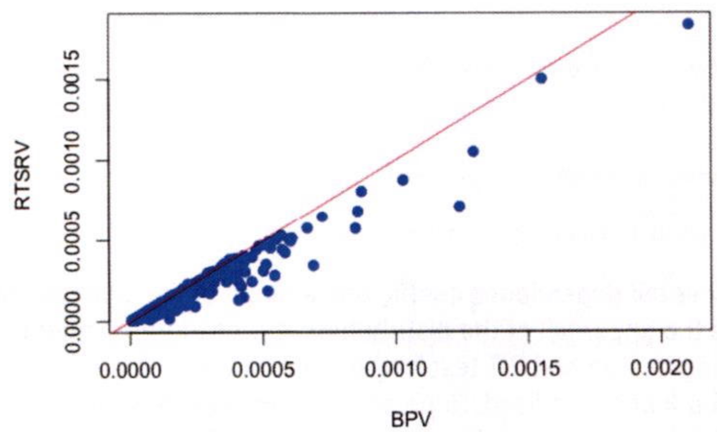
Ivana Filipovića 4 • 51000 Rijeka
tel +385 51 355 111
fax + 385 51 212 268
http://www.efri.uniri.hr
e-mail efri@efri.hr

showing the positive relationship between $RTSRV_t^{\Delta,k,\theta}$ and two other realized volatility estimators. They present how well the bipower variation (BPV_t^{Δ}) and average subsampled realized variance ($ARV_t^{\Delta,k}$) fit to robust two times scaled realized variance. Scatter plot indicates that both bipower variation (BPV_t^{Δ}) and average subsampled realized variance ($ARV_t^{\Delta,k}$) underestimate the variance i.e. benchmark.

Table 5 Results of Mincer-Zarnowitz regression

	$MedRV_t^{\Delta}$	$MinRV_t^{\Delta}$	RV_t^{Δ}	BPV_t^{Δ}	$TSRV_t^{\Delta,k}$	$ARV_t^{\Delta,k}$	$HYRV_t^{\Delta}$
MIB							
test statistic	3449.3***	2298***	8375.1***	5442.6***	8841.3***	7530***	8274***
β_0	0.000002	0.000002	0.000008	0.000001	0.000007	0.000007	0.000008
β_1	0.779631	0.799547	0.508353	0.739918	0.504905	0.518399	0.508959
DAX							
test statistic	2281.7***	1853.3***	6517.3***	3404.7***	8001.1***	6732.8***	6517.1***
β_0	-0.000002	-0.000001	-0.000008	-0.000003	-0.000009	-0.000009	-0.000008
β_1	0.850240	0.856405	0.766997	0.820539	0.763453	0.782201	0.766998
CAC							
test statistic	2359.3***	3739***	4611.3***	2776.1***	1853.2***	679.25***	470.67***
β_0	0.000010	0.000011	0.000004	0.000008	0.000002	0.000005	0.000005
β_1	1.275828	1.426721	1.006286	1.355853	0.792073	1.012190	1.006807
FTSE							
test statistic	4296***	4039.8***	7845.9***	5365.1***	9819.5***	8446.9***	7845.7***
β_0	0.000001	0.000001	-0.000002	-0.000001	-0.000002	-0.000002	-0.000003
β_1	0.782853	0.777687	0.724623	0.769927	0.713995	0.730009	0.724622

Note: ***, **, * represent significance of the chi-squared test at the 1%, 5% and 10% level.



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EKONOMSKI FAKULTET

Ivana Filipovića 4 • 51000 Rijeka
tel +385 51 355 111
fax + 385 51 212 268
http://www.efri.uniri.hr
e-mail efri@efri.hr

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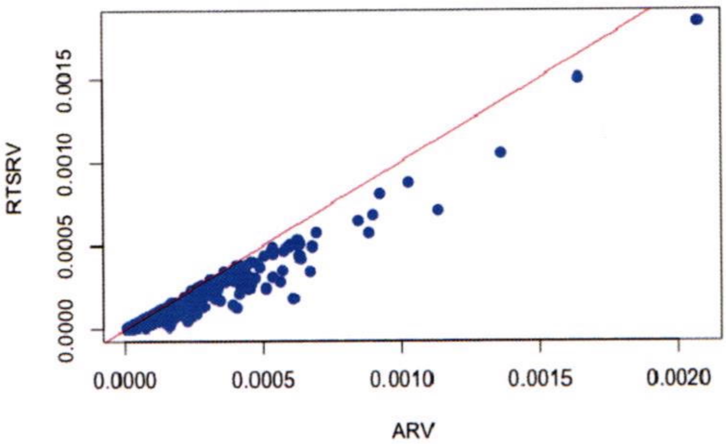
(a) Bipower variation



(b) Average subsampled realized variance

Figure 21 Relationship between robust two times scaled realized variance and other realized volatility estimators

The probability integral transformation (PIT) test was used to check whether the difference between $RTSRV_t^{\Delta,k,\theta}$ and other competing volatility estimators is uniformly distributed. The results, including Kolmogorov-Smirnov test statistic for uniformity and p-value, are presented in Table 6 below. For all European markets and all volatility estimators the null hypothesis was rejected at significance level of 5%.

Table 6 Results of PIT test

	$MedRV_t^{\Delta}$	$MinRV_t^{\Delta}$	RV_t^{Δ}	BPV_t^{Δ}	$TSRV_t^{\Delta,k}$	$ARV_t^{\Delta,k}$	$HYRV_t^{\Delta}$
MIB							
test statistic	14399.1***	16639.3***	9577.5***	14373.9***	8801.2***	10161.2***	9554.1***
DAX							
test statistic	15123.4***	15272.9***	12953.1**	16349.2**	10790.7***	11598.8***	12955***
CAC							
test statistic	10636**	10152***	11580**	10692***	17964***	11624***	11579***
FTSE							
test statistic	20727**	12495***	13824**	18095***	14016***	13099***	13824***

Note: ***, **, * represent significance of the Kolmogorov-Smirnov test at the 1%, 5% and 10% level.

This research utilizes the upper tail dependence coefficient, a result of the Gumbel copula function. When the focus of interest is the upper tail of the distribution, it is used for comparison purposes. While the Mincer-Zarnowitz regression and PIT test haven't shown the preference of a specific estimator, the upper tail dependence is utilized. Representing the extreme value distributions, the Gumbel copula function is an upper tail dependence measure given by:

SVEUČILIŠTE U RIJECI
EKONOMSKI FAKULTET

Ivana Filipovića 4 • 51000 Rijeka
tel +385 51 355 111
fax + 385 51 212 268
http://www.efri.uniri.hr
e-mail efri@efri.hr

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$$C(u,v) = \exp\left\{-\left[-\ln(u)^{\delta} + \left(-\ln(v)^{\delta}\right)\right]^{\frac{1}{\delta}}\right\}$$

where $u, v \in [0,1]$, with parameter $1 \leq \delta \leq \infty$ that controls the strength of reliance. Upper tail dependence is a function of Gumbel copula parameter $\lambda_u = 2 - 2^{\frac{1}{\delta}}$.

Table 7 Upper tail dependence results

	$MedRV_t^{\Delta}$	$MinRV_t^{\Delta}$	RV_t^{Δ}	BPV_t^{Δ}	$TSRV_t^{\Delta,k}$	$ARV_t^{\Delta,k}$	$HYRV_t^{\Delta}$
MIB							
δ	7.09	6.81	5.35	6.72	5.23	5.51	5.37
λ	0.859	0.853	0.813	0.851	0.809	0.819	0.814
DAX							
δ	7.96	7.65	6.51	7.6	6.64	6.65	6.51
λ	0.874	0.869	0.846	0.868	0.849	0.85	0.846
CAC							
δ	5.86	5.72	5.75	5.93	6.93	5.85	5.75
λ	0.829	0.825	0.826	0.831	0.856	0.829	0.826
FTSE							
δ	7.34	7.02	5.69	7.06	5.81	5.8	5.69
λ	0.864	0.858	0.824	0.858	0.828	0.828	0.824

The results given in Table 7 present the tail dependence coefficient λ based on the Gumbel copula function and δ that regulates the degree of reliance. The results indicate that for Italy, Germany and UK, $RTSRV_t^{\Delta,k,\theta}$ has the highest upper tail dependence with $MedRV_t^{\Delta}$, $MinRV_t^{\Delta}$ and BPV_t^{Δ} volatility estimators. Among all the competing estimators, only jump robust ones have produced almost similar volatility estimates as $RTSRV_t^{\Delta,k,\theta}$ (Andersen et al., 2012; Barndorff-Nielsen & Shephard, 2006). In case of France, $RTSRV_t^{\Delta,k,\theta}$ is best fitted with $TSRV_t^{\Delta,k}$, BPV_t^{Δ}, $ARV_t^{\Delta,k}$ and $MedRV_t^{\Delta}$ volatility estimators. As $TSRV_t^{\Delta,k}$ is robust to microstructure noise, it is of no surprise that estimates are as good as $RTSRV_t^{\Delta,k,\theta}$. Due to inconclusive results from Mincer-Zarnowitz and PIT test, the upper tail dependence was introduced because it examines the events in tails i.e. extreme values. The results indicated that the medianized block of three returns ($MedRV_t^{\Delta}$) performed most similar to the robust two times scaled realized variance ($RTSRV_t^{\Delta,k,\theta}$) for Italy, Germany and UK. For France the two times scaled realized variance ($TSRV_t^{\Delta,k}$) realized volatility estimator was the most similar (approximately equal) to the benchmark. Since medianized block of three returns ($MedRV_t^{\Delta}$) is robust only to price jumps, we conclude that the Italian, German and UK financial markets are more contaminated by price jumps than by microstructure noise at selected sampling frequencies. The French financial market is more

Ivana Filipovića 4 • 51000 Rijeka
tel +385 51 355 111
fax + 385 51 212 268
http://www.efri.uniri.hr
e-mail efri@efri.hr

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contaminated by microstructure noise than by price jumps.

This contributes to the existing literature in several ways. The main finding considers the selection of optimal slow time scale frequency in favor of two times scaled estimator in each market individually, at the same time ensuring robustness to price jumps. This research contributes to the previous studies with an empirical dataset consisting of high-frequency price observations comprising four main European market indices (DAX, CAC, FTSE and MIB), because there are very few studies that take into account the calculations of realized volatility estimators on developed European markets. Another novelty is the usage of a combination of three tests for benchmarking: Mincer-Zarnowitz regression, PIT test and upper tail dependence test within the Gumbel copula where the results are given for each of the observed developed European markets. The results are important to financial analysts and investors because they offer a recommendation which realized volatility estimator to use for the observed market indices. An additional contribution is also a determined optimal sampling frequency for each of the observed developed European markets.

4. Benefits of sectoral cryptocurrency portfolio optimization.

In the third part of this dissertation will formally identify and describe the benefits of sectoral cryptocurrency classification portfolio optimization and it's performance. Six optimization targets will be formed: MinVar, MinCVaR, MaxSR, MaxSTARR, MaxUT and MaxMean. We compare the obtained portfolios with the performance of the CRIX index (representing the crypto market) over the same period. Our results show that five of the six portfolio strategies performed better if they included sectoral cryptocurrencies namely from financial, exchange and business services sectors.
For the purpose of this study, we used publicly available daily price data (in USD) for a total of 65 cryptocurrencies collected from the Coinmarketcap - CMC platform pages, was used. Data was collected for the period from 8/26/2019 to 02/22/2020 creating a sample of a total of 146 daily observations, or 145 daily returns for 65 time series. To test the utility of cryptocurrency sectoral division, an existing portfolio consisting of the top 50 cryptocurrencies by market capitalization includes additional 15 cryptocurrencies, 5 leading cryptocurrencies by each of the three leading utilization sectors by market capitalization: finance, exchanges and business services. Sectoral cryptocurrencies that entered the first 50 by market capitalization were excluded and replaced by the next utilization token by size of market capitalization in the respective sector. We form multiple portfolios with different optimization goals of risk minimization, return maximization and maximization of return and risk ratios. Given the results of previous research by Briere et al. (2015) and Lee Kuo Chuen et al. (2018) and the absence of a normal distribution of returns, apart from the standard deviation, will use the conditional Value at Risk - CVaR for the risk measure, i.e. the methodology that follows the work of (Rockafellar and Uryasev, 2000), with a confidence level of 95%. Our optimization goals are as follows: minimum variance (MinVar), minimum CVaR (MinCVaR), maximize sharpe ratio (MaxSR), maximize stable tail-adjusted return ratio (MaxSTARR), maximize utility function (MaxUT) and maximize mean return (MaxMean). In order to examine the benefits of treating the cryptocurrency market through sector division we conduct the research in two steps. The first step is to form and test the performance of a portfolio whose components make up the first 50 cryptocurrencies by market capitalization. In the second step, an additional 15 sectoral cryptocurrencies are included in the existing data set. In order to achieve the inclusion of sector cryptocurrencies in the portfolio, in the second step, linear group constraints are created where 20% of the total portfolio allocation must be allocated to sector cryptocurrencies according to the optimization goals. The notation of portfolio

SVEUČILIŠTE U RIJECI
EKONOMSKI FAKULTET

Ivana Filipovića 4 • 51000 Rijeka
tel +385 51 355 111
fax + 385 51 212 268
http://www.efri.uniri.hr
e-mail efri@efri.hr

optimization goals involving sector cryptocurrencies is as follows: minimum variance-sector (MinVar-S), minimum CVaR-sector (MinCVaR-S), maximize sharpe ratio-sector (MaxSR-S), maximize stable tail-adjusted return ratio- sector (MaxSTARR-S), maximize utility function-sector (MaxUT-S) and maximize mean return-sector (MaxMean-S). Optimization is performed out of sample (backtesting), with the same parameters for each optimization goal. A time period of $k = 10$ days was used to estimate the initial parameters and portfolio allocation. Given the dynamics of the cryptocurrency market, a more frequent monthly rebalance of $K = 30$ days was chosen with the so-called extending window approach $k + K$. For each period $k + 1$, portfolio returns are drawn with respect to the results of the allocation optimization in the previous k, i.e. $k + K$ moment.
We present and interpret the obtained empirical results in two phases. In the first phase, the results are reviewed and interpreted by a comparative method between asset allocation models according to the initial selection of the portfolio components. In addition, the success of a particular strategy is judged by the implementation of performance measures that include the CRIX index as a benchmark for the crypto market over the observation period. In second phase, the results of allocation models are compared and interpreted between portfolios to determine the benefits of dividing and optimizing cryptocurrencies according to their appropriate sectors. Table 8 shows the results of the previously described performance measures for six asset allocation models for 50 cryptocurrencies selected by CMC market size. Table 2 shows the results of portfolio performance measures that, in addition to the 50 cryptocurrencies per CMC, include an additional 15 cryptocurrencies per related financial, exchange and business services sector. The last column in the tables shows the results of the CRIX index over the same period as the benchmark of the cryptocurrency market. All values except the regression beta and worst drawdown of each optimization strategy are reported annually. In case of negative values the Traynor ratio is omitted. If the regression beta between portfolio returns and the CRIX index is considered in the context of the CAPM model, most portfolios have a very low systematic risk. Moreover, four strategies have achieved a negative beta, which means that they are moving in the opposite direction of the CRIX index. Positive regression alphas indicate that, in the case of a cryptocurrency market stagnation, each of the observed portfolios on average achieves higher returns than the market returns. As expected, the highest average alpha was achieved by the MaxMean portfolio. On the other hand, it is worth pointing out that the highest annual geometric return as well as the cumulative return in the total period, was achieved by portfolio with the optimization goal of minimizing CVaR. Therefore we conclude that the best values of the performance measures are expected to be related to the MinCVaR portfolio. So SR stands at a high of 2,68 which is by far the best of all other optimization strategies. The difference between the MSquared return and the risk ratio relative to the CRIX index also benefits the portfolio, which minimizes the conditional value at risk. Jensen's alpha which suggests if the strategy has outperformed the market for all optimization solutions is positive. In other words, all strategies yielded returns higher than required by the CAPM model. The highest ratio of active premium IR and standard deviation was also achieved by the MinCVaR portfolio.

Table 8. Asset allocation models without sectors cryptocurrencies

		Asset Allocation Models						
Performance Metrics		MinVar	MinCVaR	MaxSR	MaxSTARR	MaxUT	MaxMean	CRIX
Beta	β_i	0,05	-0,05	0,02	-0,014	-0.05	-0,01	1
Annualized Alpha	α_{ai}	1,12	1,91	1,16	1,53	0,97	2,29	/
Annualized	R_{Ci}	0,94	1,44	0,95	0,62	0,72	0,95	0,57

SVEUČILIŠTE U RIJECI
EKONOMSKI FAKULTET

Ivana Filipovića 4 • 51000 Rijeka
tel +385 51 355 111
fax + 385 51 212 268
http://www.efri.uniri.hr
e-mail efri@efri.hr

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Return								
Annualized Std Dev	σ_{ai}	0,49	0,54	0,48	0,94	0,46	1,04	0,47
Worst Drawdown	WD	0,27	0,29	0,26	0,57	0,27	0,56	0,31
Cumulative Return	CY	1,46	1,67	1,47	1,32	1,37	1,47	1,29
Sharpe Ratio	SR	1,92	2,68	1,95	0,66	1,58	0,91	1,20
MSquared	M^2	0,91	1,27	0,92	0,31	0,75	0,43	0,57
Treynor Ratio	TR	18,69	/	59,03	/	/	/	0,57
Jensen's Alpha	α_i	0,91	1,47	0,94	0,63	0,75	0,95	/
Information Ratio	IR	0,56	1,20	0,56	0,05	0,23	0,34	/

On the other hand, the lowest geometric and cumulative returns were achieved by an optimization strategy for maximizing the ratio of returns and CVaR, so the values of other performance measures for MaxSTARR strategy are also lower than performance measure of other strategies. The highest annual standard deviation was achieved by the portfolio with the optimization goal of maximizing expected returns, and the lowest portfolio maximizing the utility function. In comparison with the CRIX index, all implemented optimization goals achieved a higher cumulative return in the same observation period. However, the CRIX index achieved a lower standard deviation level in five out of six observed cases. Four portfolios have smaller worst drawdowns than the index as well as a higher SR. Figure 22 shows the dynamics of the daily cumulative returns of individual strategies, the total daily returns of all strategies, and an underwater chart for drawdown to further illustrate the performance of portfolio optimization goals.

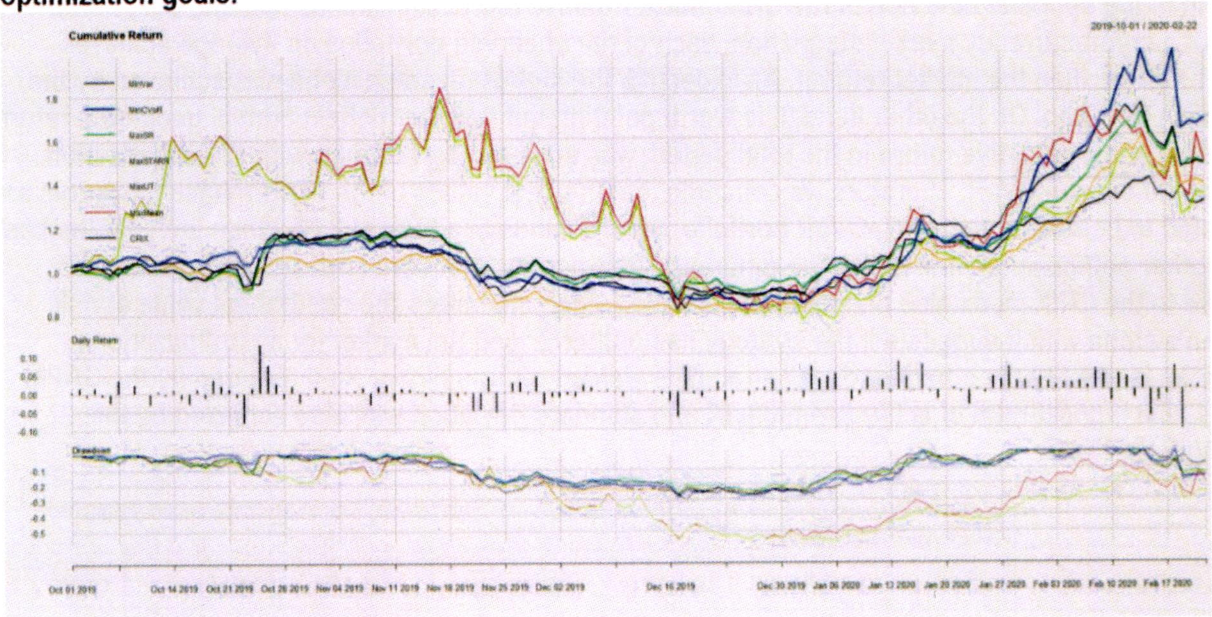

Fig. 22. Performance summary various strategies without sectors cryptocurrencies

By including an additional 15 sectoral cryptocurrencies that would not initially be selected as a

Ivana Filipovića 4 • 51000 Rijeka
tel +385 51 355 111
fax + 385 51 212 268
http://www.efri.uniri.hr
e-mail efri@efri.hr

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component of the portfolios by their market capitalization, the results differ by all measures shown in Table 9. Similar to the above, the regression beta values suggest low systematic risk of all optimization strategies. The highest average annual alpha, geometrical as well as the total cumulative return, was achieved by the portfolio with the aim of maximizing it. Which is in line with expectations due to the higher risk assumed as standard deviation, i.e. worst drawdown. However, the return of the MaxMean-S portfolio adequately compensated for the higher risk assumed, which ultimately resulted in a high SR of 4,66. MSquared also points out the difference between the MaxMean-S portfolio and the CRIX index. Jensen's alpha suggests that all observed portfolios have yielded higher than expected returns per CAPM ratio. The significant difference between the realized geometric return of the MexMean-S portfolio and the CRIX index, comparing to the standard deviation of the active premium which is extremely low due to the equal volatility between the observed investments, also influenced the highly positive Information ratio. In the second order of the best size of all performances, except for the risk measures, it was achieved by a portfolio that maximizes the ratio of return and risk expressed as CVaR.

Table 9. Asset allocation models with sectors cryptocurrencies

		Asset Allocation Models						
Performance Metrics		MinVar-S	MinCVaR-S	MaxSR-S	MaxSTARR-S	MaxUT-S	MaxMean-S	CRIX
Beta	β_i	0,05	0,03	-0,08	0,10	-0,12	0,22	1
Annualized Alpha	α_{ai}	0,86	2,39	1,37	3,74	1,51	10,10	/
Annualized Return	R_{Gi}	0,73	1,99	1,09	3,02	1,09	5,84	0,57
Annualized Std Dev	σ_{ai}	0,45	0,53	0,43	0,67	0,48	1,17	0,47
Worst Drawdown	WD	0,27	0,22	0,29	0,23	0,25	0,35	0,31
Cumulative Return	CY	1,37	1,88	1,52	2,23	1,53	3,02	1,29
Sharpe Ratio	SR	1,62	3,79	2,53	4,50	2,27	4,99	1,20
MSquared	M^2	0,77	1,79	1,20	2,13	1,07	2,36	0,57
Treynor Ratio	TR	14,91	76,54	/	31,22	/	26,88	0,57
Jensen's Alpha	α_i	0,70	1,98	1,12	2,97	1,16	5,72	/
Information Ratio	IR	0,26	2,04	0,77	3,09	0,74	4,31	/

In terms of performance measures, the strategy to minimize standard deviation of the portfolio has performed the worst. On the other hand, the lowest standard deviation was achieved by the MaxSR-S optimization strategy, where the standard deviation of the MinVar-S portfolio is slightly higher. The lowest worst drawdown belongs to the MinCVaR-S portfolio, which is in line with the optimization goal. In comparison with the CRIX index, all of the optimization goals achieved a higher cumulative return in the same observation period. It is also worth pointing out that only two MaxUT-S and MaxMean-S strategies achieved a higher standard deviation than the CRIX index. In addition, all portfolios achieved a higher Sharpe ratio than the CRIX index during the same period. Figure 23 shows the dynamics of the daily cumulative returns of individual strategies, the total daily returns of all strategies, and the underwater chart for drawdown, further illustrating the performance of portfolio optimization goals.

SVEUČILIŠTE U RIJECI
EKONOMSKI FAKULTET

Ivana Filipovića 4 • 51000 Rijeka
tel +385 51 355 111
fax + 385 51 212 268
http://www.efri.uniri.hr
e-mail efri@efri.hr

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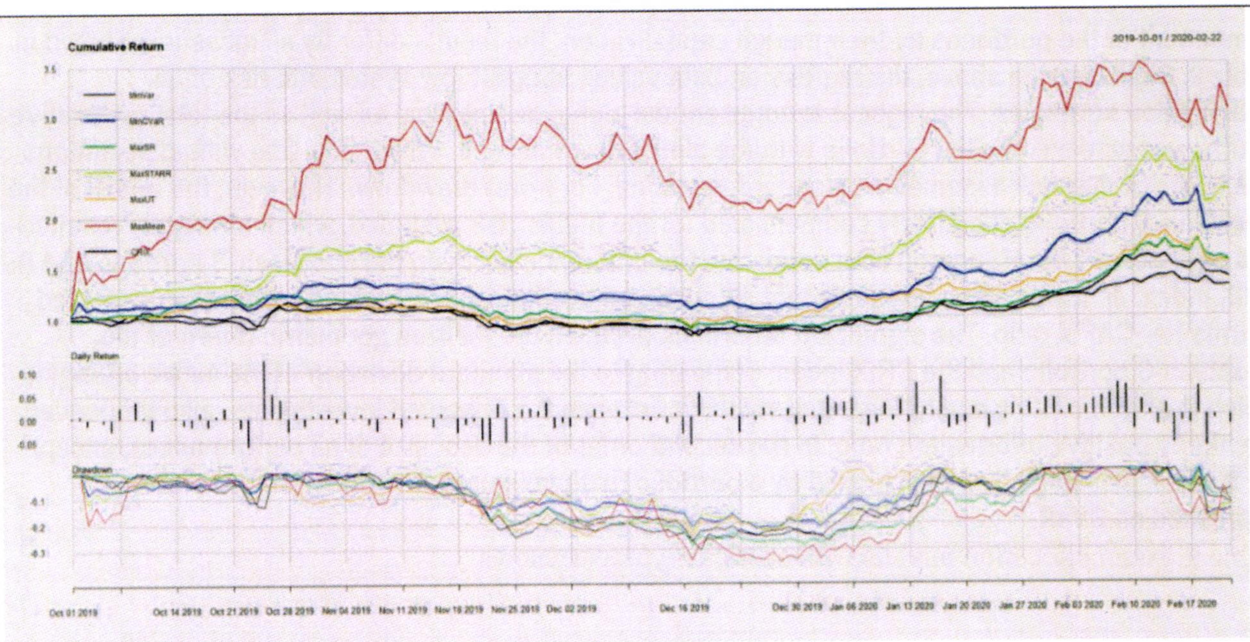

Fig. 23. Performance summary various strategies with sectors cryptocurrencies

Interpreting the results involves considering the performance of strategies between portfolios that differ in composition. The first thing to notice is the height of the regression alpha, which for all portfolios except the MinVar-S, achieved a better result if sectoral cryptocurrencies were included in the portfolio. Portfolios with additional cryptocurrencies earn on average more returns than portfolios without sectoral components. Geometric return has the same relationships. Only the MinVar-S portfolio has a lower return than the portfolio return without sectoral cryptocurrency, thus confirming our finding and logic that there are benefits in treating the cryptocurrency market through sectoral affiliation. In terms of risk, four strategies involving sectoral cryptocurrencies have achieved a lower standard deviation than portfolios without them. However, the higher risk was offset by the higher return achieved, implying a higher Sharpe ratio. Worst drawdown also points to the benefits of including additional cryptocurrencies where only the MaxSR-S portfolio has a higher worst drawdown than the MaxSR portfolio. The inclusion of sector cryptocurrencies has also led to an increase in cumulative return for all strategies except the MinVar-S portfolio. The biggest difference was recorded by the MaxMean-S portfolio, where its cumulative return increased by 1,55. By applying a return maximization strategy and considering sectoral cryptocurrencies as a components of the portfolio, it was possible to achieve a cumulative return higher by 105% over a 146-day period than the same strategy that does not consider sectoral cryptocurrencies. A significant increase in cumulative return was also achieved by the MaxSTARR-S portfolio of 0,91, or 69%, compared to MaxSTARR. The results of the Sharpe ratio and MSquared measures are consistent with the above findings. The largest increase in SR and〖 M〗^2 refers to the MaxMean-S portfolio. Only the MinVar-S portfolio has a lower SR and M^2 relative to an equivalent strategy with no additional cryptocurrencies. Given that five of the six portfolios that include sectoral cryptocurrencies had a higher geometric return and the regression beta did not increase significantly, so Jensen's alpha performed better for all portfolios except MinVar-S. The inclusion of additional sector cryptocurrencies in existing portfolios contributes to the improvement of portfolio performance compared to the market represented by the CRIX index. Treynor ratio and Information ratio also performed significantly better for all sector portfolios except MinVar-S portfolios. Considering all the above, it can be concluded that five of the six portfolios

SVEUČILIŠTE U RIJECI
EKONOMSKI FAKULTET

Ivana Filipovića 4 • 51000 Rijeka
tel +385 51 355 111
fax + 385 51 212 268
http://www.efri.uniri.hr
e-mail efri@efri.hr

created according to different optimization goals achieved better results if they view the cryptocurrencies through the sectoral perspective (financial, exchange and business services). Such results contribute significantly to the research of investment opportunities in the cryptocurrency market and sectoral segmentation of cryptocurrency market. In addition, the positive results point to two more observations from which certain conclusions need to be drawn. The first observation is certainly the existence of distinction within the category of cryptocurrencies and also the category of utilization tokens. If cryptocurrency litecoin is used solely as a means of payment, comparing litecoin with a decentralized computer platform like ethereum and treating the two assets equally in the context of investment opportunities, is simply not practical or even impossible and this also confirmed by our results. On blockchain economies like ethereum, among other things, tokens as a type of cryptocurrency, can be created to provide a different purpose and utility in the practical application of the product and service. This also means that products and services are different, i.e. utilization tokens differ in their fundamentals. For an example, a utilization token with a strictly defined purpose in a specific online game should not be viewed as equal with a utilization token that has a wider purpose, such as tokens created for decentralized finances. If viewed together, the market recognizes such anomalies and "properly" values the assets observed. This situation has led to significantly better portfolio results when including sectoral cryptocurrencies. In the cryptocurrency market beginning of 2020 and in 2019, was marked by a rise in the value of utilization tokens, which to some extent represented Decentralized Finance - DeFi. None of the 15 additional cryptocurrencies (tokens) selected by sector were in the top 50 by CMC, so they have not been considered in previous studies. Previous research papers have considered cryptocurrencies as a homogenous asset and relied solely on the general market optimization algorithm when selecting portfolio components. Such an approach implies a consensus on the magnitude of the equilibrium expected return of the selected cryptocurrencies. However, previously it has been shown that such a return does not even exist due to the absence of adequate valuation, that is, the intrinsic value of cryptocurrencies. Taking into account previous research, if one draws a parallel with thinking of the traditional capital market and the CAPM model, it can be concluded that all cryptocurrencies are properly valued, i.e. all cryptocurrencies are on the Security Market Line-SML. Our results in this paper suggest the opposite. Our conclusions are supported by positive average regression and realized alpha portfolios, as well as portfolios with additional sectoral cryptocurrencies. In line with the obtained results, our findings emphasize the utility and necessity of observing the cryptocurrency market by sectoral affiliation with the aim of finding potentially "undervalued" cryptocurrencies. If portfolio components are selected solely by market capitalization, it would mean that these cryptocurrencies have already achieved the value that makes them a potential portfolio component. The possibility of price growth of such cryptocurrency is certainly much lower than the possibility of cryptocurrency growth which ranks much lower in terms of market capitalization. Sectorally, cryptocurrencies with much lower market capitalization are emerging and investors can more easily spot them. Looking at the overall capitalization of the sector, it is easier to spot and identify current trends in the cryptocurrency market, as was the growth trend in 2019 of DeFi cryptocurrencies.

The results suggest that portfolios in which 20% of the weight is allocated to cryptocurrencies of lower market capitalization achieve higher values across all implemented performance measures in five of the six optimization strategies. It can be concluded that it is desirable and necessary to observe the cryptocurrency market through their type or their utility, and such an approach can be achieved by categorizing cryptocurrencies into their sectors. Potential investors, and portfolio managers in particular, should not consider cryptocurrencies only based on their market capitalization. Cryptocurrencies have characteristics and capabilities that define them according to their nominal

SVEUČILIŠTE U RIJECI
EKONOMSKI FAKULTET

Ivana Filipovića 4 • 51000 Rijeka
tel +385 51 355 111
fax + 385 51 212 268
http://www.efri.uniri.hr
e-mail efri@efri.hr

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purpose. Accordingly, portfolio managers are encouraged to consider cryptocurrencies by their characteristics (the type and purpose they provide) when constructing a portfolio, in order to eliminate their subordinate position and to contribute to portfolio performance in the cryptocurrency market.

The main objective of our research was to identify and describe the construction of a portfolio created towards different optimization goals. The constituents of these portfolios are cryptocurrencies as a new type of an asset and for the first time categorized in sectors to be included as potential candidates for portfolio construction. Presented results suggest that 20% weight of portfolio allocated according to sectoral classification of cryptocurrencies, provides the opportunity to achieve better portfolio performance measures in five of the six optimization strategies. In our research it has been concluded that sectoral classification identifies cryptocurrencies with higher potential of market growth as well as increasing diversification and reducing portfolio risk. This is a novelty and enriches recent literature on rapid and fast paced growing topic of cryptocurrencies. It is valuable to investors and regulators, though especially for the portfolio managers, during the practical implementation of our research that suggests considering cryptocurrencies as potential constituents in portfolios through their type and value.

5. Putting safe haven currencies to the test using benchmark realized covariance estimator.

In the fourth and last part of the dissertation we examined three FX currencies (U.S. Dollar, Swiss Franc, and Japanese Yen) and Bitcoin in a 1-minute time window from June 2013 to May 2022. The study was conducted in two phases. First, simulation designs on high-frequency data were used to determine which realized covariance estimator performs best. Second, the best covariance estimator, the robust two times scaled estimator, is applied to real high-frequency data, comparing observed currencies to the equity market. This research thus makes two contributions to the literature. First, it determines the best covariance estimator and second, it identifies the best performing safe haven currencies compared to general market movements. In the current literature, there is no strict consensus on the superiority of the realized covariance estimator, so the results of this research paper will make an important contribution because they will suggest which estimators are appropriate for each analyzed market and how to solve the problem of microstructural noise, price jumps and non-synchronization, which are closely related to the selection of the optimal sampling frequencies on both the slow and fast time scale. There is no consensus on which asset is better safe haven, especially with respect to cryptocurrencies such as Bitcoin, as stated in Wen (2022). We will provide an answer in this research. One of the main objectives of this study is to determine the best covariance estimator for both synchronized and unsynchronized high frequency data. According to the results of the simulation study, the best covariance estimator is used for real high frequency data set. The specific objectives of this study are:

a. To examine the covariance estimators ($rBPCov_t^{\Delta}$, $rCov_t^{\Delta}$, $rHYCov_t^{\Delta,\theta}$, $rThresholdCov_t^{k,h}$, $rTSCov_t^{\Delta,k}$, and $rRTSCov_t^{\Delta,k,\theta}$) for both synchronized and unsynchronized high-frequency data.
b. To compare the covariance estimators' performances basing on measures of fit i.e. Relative bias and Root mean squared error (RMSE) for both simulations of

SVEUČILIŠTE U RIJECI
EKONOMSKI FAKULTET

Ivana Filipovića 4 • 51000 Rijeka
tel +385 51 355 111
fax + 385 51 212 268
http://www.efri.uniri.hr
e-mail efri@efri.hr

synchronized and unsynchronized high-frequency data.

c. To examine the impact of jumps on the model on all of the six covariance estimators for both synchronized and unsynchronized high-frequency data.

d. Using the obtained results on covariance estimators on real world high-frequency data we explore which type of an asset is a better safe haven. Negative correlation with general market indicates safe haven characteristics as in Kaczmarek (2022).

The first objective of the present paper is to explore and determine the most effective covariance estimator for high-frequency datasets. This includes both synchronized and unsynchronized data. The study aims to provide comprehensive and reliable insights into optimal approach for estimating covariance in high-frequency data analysis. Our research presents rigorous and structured two-phase approach to evaluate the measuring accuracy of each estimator. Firstly, we employ a simulation design to provide a comprehensive assessment of the accuracy of each estimator. This approach ensures that the results are not biased and are reliable. Secondly, we utilize the conclusions drawn from the simulation results to analyze high-frequency real-world data. This approach enables us to gain a practical understanding of the performance of each estimator in real-world scenarios. To ensure the accuracy of our simulations, we follow the methodology similar to the one proposed in Boudt and Zhang (2015) and Barndorff-Nielsen et al. (2011). We simulate for a period of $S = 100$ days with 1 second increment between finite data samples. Figure 24 shows 10 day subsample of the simulated data, providing a visual representation of how the data is structured and how each estimator is evaluated. The simulation design enables us to compare the performance of each estimator in different scenarios and to identify any potential biases or limitations in their accuracy. This approach ensures that the results of our research are robust and reliable.

$$
\begin{aligned}
d\tilde{X}_{1t} &= \gamma_{x_1}\sigma_t^{x_1}dB_t^{x_1} + \sqrt{1-\gamma_{x_1}^2}\,\sigma_t^{x_1}dW_t + dZ_t^{x_1},\\
d\tilde{X}_{2t} &= \gamma_{x_2}\sigma_t^{x_2}dB_t^{x_2} + \sqrt{1-\gamma_{x_2}^2}\,\sigma_t^{x_2}dW_t + dZ_t^{x_2}, \qquad (1)\\
\sigma_t^{x_1} &= e^{(\beta_0+\beta_1 v_t^{x_1})}, \qquad dv_t^{x_1} = \alpha v_t^{x_1}dt + dB_t^{x_1},\\
\sigma_t^{x_2} &= e^{(\beta_0+\beta_1 v_t^{x_2})}, \qquad dv_t^{x_2} = \alpha v_t^{x_2}dt + dB_t^{x_2}
\end{aligned}
$$

In this study, we are dealing with a system of stochastic differential equations. The equations are represented by $\sigma_t^{x_1} = e^{(\beta_0+\beta_1 v_t^{x_1})}$ and $dv_t^{x_1} = \alpha v_t^{x_1}dt + dB_t^{x_1}$ for X_1 as well as $\sigma_t^{x_2} = e^{(\beta_0+\beta_1 v_t^{x_2})}$ and $dv_t^{x_2} = \alpha v_t^{x_2}dt + dB_t^{x_2}$ for X_2. We use Boudt and Zhang (2015) assumptions that include $B_{X_1} \perp B_{X_2}, B_{X_1} \perp W$, and $B_{X_2} \perp W$. Moreover, the parameters $(\beta_0, \beta_1, \alpha, \gamma_{x_1}, \gamma_{x_2})$ are set to $(\frac{5}{16}, \frac{1}{8}, \frac{1}{40}, 0.3, 0.3)$. The initial value of $v_t^{x_1}$ and $v_t^{x_2}$ for each day is drawn from je normal distribution $N(0, \frac{-1}{2\alpha})$. The spot correlation between the continuous part of the log-price changes is $\rho = 0.91$. To simulate the independent noise, we assume that $\varepsilon_t^{X_i} \sim N(0, \sigma_t^{x_i}), i = 1,2$. The system of differential equations can be simulated using the Euler scheme with an increment of 1 second per tick, as described in Boudt and Zhang (2015). It is worth noting that we aim to simulate the behavior for $X_{it} = \tilde{X}_{it} + \varepsilon_t^{X_i}, i = 1,2$.

SVEUČILIŠTE U RIJECI
EKONOMSKI FAKULTET

Ivana Filipovića 4 • 51000 Rijeka
tel +385 51 355 111
fax + 385 51 212 268
http://www.efri.uniri.hr
e-mail efri@efri.hr

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In order to find a solution for the system of differential equations presented in equation (1) we rely on the variables X_{1t} and X_{2t}. One method for obtaining a solution through the application of Ito's lemma. This technique allows us to rewrite the original equations in a way that is more amenable to finding a solution. By utilizing this approach, we can arrive at solution that is both accurate and reliable.

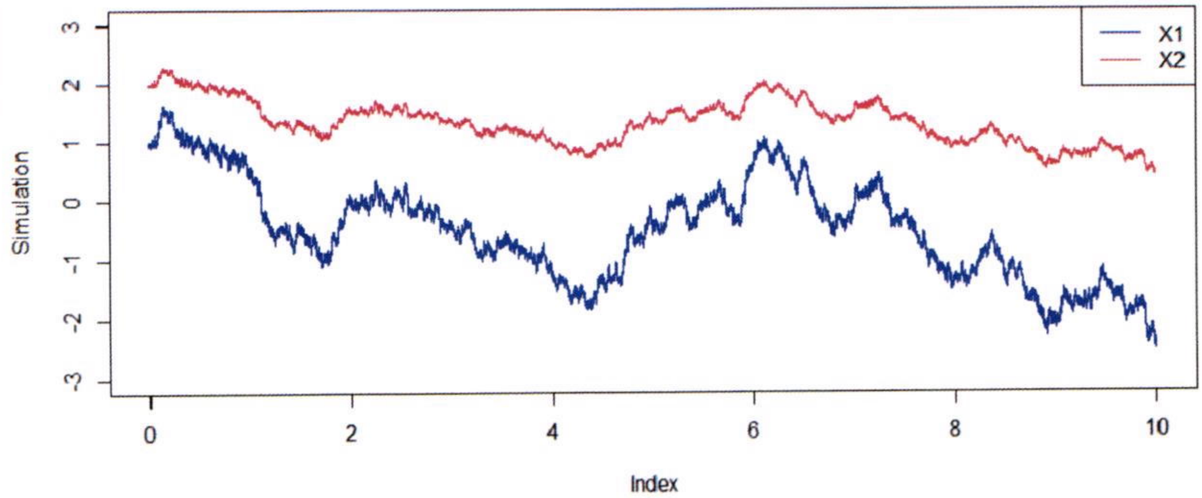
$$\tilde{X}_{it} = \tilde{X}_0 + \gamma_{x_i}\sigma_t^{x_i}B_t^{x_i} + \sqrt{1-\gamma_{x_i}^2}\sigma_t^{x_i}W_t + Z_t^{x_i}, i = 1,2 \qquad (2)$$

with $B_t^{x_i} \sim N(0,t), W_t \sim N(0,t)$ and $Z_t^{x_i} \sim \text{Poisson}(t), Z_0^{x_i} = 0, i = 1,2$.

In order to replicate the values of $X_{it}, i = 1,2,$ we employ several stochastic processes. Specifically, we utilize $B_t^{x_i}, i = 1,2,$ which conforms to a normal distribution characterized by mean of zero and a standard deviation equal to the square root of t. Additionally, we incorporate W_t which is also a normal distribution with a mean of zero and a standard deviation of t. Finally, we consider $Z_t^{x_i}, i = 1,2$ which follows a Poisson distribution with parameter value of t. It is important to note that $Z_0^{x_i}, i = 1,2$ is set to zero as an initial condition. By leveraging these stochastic processes, we are able to effectively simulate $X_{it}, i = 1,2$ with high degree of accuracy.

$$X_{it} = \underbrace{\tilde{X}_0 + \gamma_{x_i}\sigma_t^{x_i}B_t^{x_i} + \sqrt{1-\gamma_{x_i}^2}\sigma_t^{x_i}W_t + Z_t^{x_i}}_{\tilde{X}_{it}} + \varepsilon_t^{X_i}, i = 1,2 \qquad (3)$$

FIG 24. Simulation of high-frequency data, 10 day subsample



In the simulation study the estimators from Čuljak et al. (2022) realized covariance version are compared to determine if the proposed $rRTSCov_t^{\Delta,k,\theta}$ has the best accuracy among alternative competitors of realized covariance. Six realized covariance estimators are examined in this research paper in order to determine which can be used as a benchmark. Threshold covariance ($rThresholdCov^{k,h}{}_t$) estimator incorporates univariate jump detection rules to mitigate the impact of jumps on the covariance estimate. By utilizing these rules, it aims to reduce the influence of jumps on the estimated covariance matrix. This approach allows the $rThresholdCov^{k,h}{}_t$ estimator to remain

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EKONOMSKI FAKULTET

Ivana Filipovića 4 • 51000 Rijeka
tel +385 51 355 111
fax + 385 51 212 268
http://www.efri.uniri.hr
e-mail efri@efri.hr

feasible even in high-dimensional settings. However, it is important to note that the estimator may be less robust to capturing small cojumps, which are simultaneous jumps or changes in multiple variables. The threshold value TR_M is set to $9\beta^{-1}$ times the daily realized bipower variation of an asset k, as suggested in Jacod and Todorov (2009), where M is the number of intraday returns. Realized covariance estimator $(rCov_t^{\Delta})$ calculates realized covariance between two assets by multiplying the realized variances of the two assets with their corresponding cross-product of returns. The estimator is robust to the microstructural noise depending on the sampling frequency on a slow and fast time scale. It is important to choose an appropriate time window or sampling frequency, as different time scales may provide different levels of information and sensitivity to noise. Price jumps contaminate the estimations with $rCov_t^{\Delta}$. Therefore modifications of the $rCov_t^{\Delta}$ have been introduced, like Realized bipower covariance estimator $(rBPCov_t^{\Delta})$. The advantage of $rBPCov_t^{\Delta}$ is that it provides a consistent estimate of the true covariance, even in the presence of price jumps. It is robust to price jumps but is still affected by microstructural noise and non-linear dependencies between asset returns. $rBPCov_t^{\Delta}$ takes into account both the sign and magnitude of the price changes as it calculates the realized bipower covariance estimate by summing the absolute product of consecutive returns for each asset h, multiplied by a constant term ($\pi/2$). In order to avoid lack of data but still remain unbiased and asymptotically consistent estimator the Two times scaled covariance estimator was introduced ($rTSCov_t^{\Delta,k}$). As seen in the equation (4) it consists of two parts. First, it represents the average of the squared values of $r_{t_{ij}h}$ over k iterations and n_t observations at time t. Second part represents a scaled version of the squared values of $r_{t_{ij}h}$ over n_t observations at time t. This term is used to adjust for bias in the estimator. Therefore it is robust to microstructural noise and by including the scaling factors in the estimator, the bias can be reduced. Due to not being jump robust the modification of $rTSCov_t^{\Delta,k}$ was introduced. The Robust two times scaled covariance estimator ($rRTSCov_t^{\Delta,k,\theta}$) is a robust version of Two times scaled covariance estimator. It is microstructural noise and jump robust. First term of the equation, as seen in (4) represents the sum of squared returns of the h-th asset, weighted by the indicator function $I_i(\theta)$, across the k historical observations. Second term represents the sum of squared returns of the h-th asset, weighted by the indicator function $I_i(\theta)$, across all n_t observations, where $\frac{n_t-k+1}{n_t k}$ factor accounts for the bias correction. Last realized covariance estimator for comparison is Hayashi-Yoshida covariance estimator ($rHYCov_t^{\Delta,\theta}$) introduced by Hayashi and Yoshida (2005). As presented in (4), the $rHYCov_t^{\Delta,\theta}$ uses an indicator function $I_i(\theta)$ in order to truncate the effects of price jumps. Due to usage of real world 1 minute high-frequency data in second phase of this research, for the simulation study, we set fast time scale at 1 and slow time scale at 20 for the $rTSCov_t^{\Delta,k}$ and $rRTSCov_t^{\Delta,k,\theta}$ realized covariance estimators. As we simulated 1 second data the fast time scale is set as 1 and slow time scale was defined by the similar design in Čuljak et al. (2022) where optimal sampling frequency on a slow time scale was evaluated and recommended.

In the first part the realized covariance estimators are analyzed based on simulated high-frequency data. The frequency of the simulated data is 1 second for $S = 100$ days. The observed scenarios are presented in the Table 1 and Table 2, respectively. First the synchronized high-frequency data is used to analyze the accuracy of the estimators. Synchronized means that the data was used immediately

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EKONOMSKI FAKULTET

Ivana Filipovića 4 • 51000 Rijeka
tel +385 51 355 111
fax + 385 51 212 268
http://www.efri.uniri.hr
e-mail efri@efri.hr

after the simulation without tempering with the frequency between two time series X_1 and X_2 using the default frequency of 1 second between the observations. The relative bias as

Relative bias $= \frac{1}{S}\sum_{i=1}^{S} \frac{\widehat{\mathrm{IC}}_i - \mathrm{IC}_i}{\mathrm{IC}_i}$ and Root mean squared error as

RMSE $= 100\sqrt{\frac{1}{S}\sum_{i=1}^{S} \left(\widehat{\mathrm{IC}}_i - \mathrm{IC}_i\right)^2}$ are calculated for each estimator in order to measure accuracy as used in Boudt and Zhang (2015) and Čuljak et al. (2022). As presented in the Table 1 the lowest relative bias had the Robust two times scaled estimator ($rRTSCov_t^{\Delta,k,\theta}$) in case when there were no jumps present in the simulated data. In the presence of jumps the values of relative bias increase showing that the accuracy of estimators decreases. The increase of bias due to jumps is apparent from the results in Table 10. It is apparent that all of the estimators are underestimating the true value of integrated covariance IC_t. When there are no jumps present in the simulated high-frequency data the results show that the bias is less present with estimators $rBPCov_t^{\Delta}$, $rCov_t^{\Delta}$, $rHYCov_t^{\Delta,\theta}$ and $rThresholdCov_t^{k,h}$. The Robust two times scaled estimator ($rRTSCov_t^{\Delta,k,\theta}$) which is robust to price jumps has shown not much of an increase in bias regardless of the intensity of the jumps in the simulated data.

Results in Table 11 indicate that the Relative bias is impacted by jumps, as Relative bias estimates increase with an increase or inclusion of jumps for both simulations of synchronized and unsynchronized high-frequency data. This result is the same with other measures of fit like Root mean squared error (RMSE) though the associated estimated for RMSE for unsynchronized high-frequency data with or without jumps do not differ very much. However, this is not a surprise since in general the Relative bias and RMSE as statistical properties and measures of fit are different quantitatively. The non-synchronously in the high-frequency data we consider is motivated by the work of Hansen et al. (2005). Hansen et al. (2005) considered two independent Poisson process sampling schemes to generate the times of the actual observations, the same simulation design we followed. There is no evidence that including jumps improves the covariance estimators in terms of lowering RMSE, as seen in both synchronized and unsynchronized high-frequency data, the covariance estimators' RMSE increased as jumps are included on the model. We would opt for other measures of fit like Multiple R-squared and Adjusted R-squared that may give more statistical evidence as we include jumps in the model. Statistically the closer the Multiple R-squared and Adjusted R-squared of the model to 1, the better the model.

We analyzed the six covariance estimators that is $rBPCov_t^{\Delta}$, $rCov_t^{\Delta}$, $rHYCov_t^{\Delta,\theta}$, $rThresholdCov_t^{k,h}$, $rTSCov_t^{\Delta,k}$, and $rRTSCov_t^{\Delta,k,\theta}$ for both synchronized and unsynchronized high-frequency data. Our simulation results of unsynchronized high-frequency data show that $rRTSCov_t^{\Delta,k,\theta}$ is the best covariance estimator since it has the smallest Relative bias for all cases (with or without jumps). This result is consistent with literature.

While for simulation results of synchronized high-frequency data, $rRTSCov_t^{\Delta,k,\theta}$ has the smallest Relative bias compared to other covariance estimators with or without jumps. This result is supportive to our hypothesis since the associated RMSE of $rRTSCov_t^{\Delta,k,\theta}$ is smaller compared to the RMSE of other covariance estimators as jumps are included in the model. These results show that $rRTSCov_t^{\Delta,k,\theta}$ is valid to be defined as a benchmark and superior to other covariance estimators.

SVEUČILIŠTE U RIJECI
EKONOMSKI FAKULTET

Ivana Filipovića 4 • 51000 Rijeka
tel +385 51 355 111
fax + 385 51 212 268
http://www.efri.uniri.hr
e-mail efri@efri.hr

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TABLE 10. Simulation results of synchronized high-frequency data

	Relative bias	RMSE
No jumps		
$rBPCov_t^{\Delta}$	-0.5462356	2.961614
$rCov_t^{\Delta}$	-0.5462497	2.961672
$rHYCov_t^{\Delta,\theta}$	-0.5462244	2.961672
$rThresholdCov_t^{k,h}$	-0.5462497	2.961672
$rTSCov_t^{\Delta,k}$	-0.1913763	2.162879
$rRTSCov_t^{\Delta,k,\theta}$	-0.1816072	2.154633
Small jumps		
$rBPCov_t^{\Delta}$	-0.6795248	3.552866
$rCov_t^{\Delta}$	-0.6796252	3.552007
$rHYCov_t^{\Delta,\theta}$	-0.6795684	3.552058
$rThresholdCov_t^{k,h}$	-0.6796252	3.552007
$rTSCov_t^{\Delta,k}$	-0.1934863	3.228447
$rRTSCov_t^{\Delta,k,\theta}$	-0.1837072	3.224062
Large jumps		
$rBPCov_t^{\Delta}$	-0.7455796	4.099145
$rCov_t^{\Delta}$	-0.7454552	4.099058
$rHYCov_t^{\Delta,\theta}$	-0.7453924	4.098981
$rThresholdCov_t^{k,h}$	-0.7454552	4.099058
$rTSCov_t^{\Delta,k}$	-0.1952669	3.329447
$rRTSCov_t^{\Delta,k,\theta}$	-0.1888425	3.325062

TABLE 11. Simulation results of unsynchronized high-frequency data

	Relative bias	RMSE
No jumps		
$rBPCov_t^{\Delta}$	-1.05273	3.321425
$rCov_t^{\Delta}$	-1.052616	3.321758
$rHYCov_t^{\Delta,\theta}$	-1.05262	3.321736
$rThresholdCov_t^{k,h}$	-1.052616	3.321758
$rTSCov_t^{\Delta,k}$	-0.2568758	2.163979
$rRTSCov_t^{\Delta,k,\theta}$	-0.2479691	2.157633
Small jumps		
$rBPCov_t^{\Delta}$	1.246548	4.112031
$rCov_t^{\Delta}$	1.246545	4.111846
$rHYCov_t^{\Delta,\theta}$	1.247246	4.111849
$rThresholdCov_t^{k,h}$	1.246545	4.111846
$rTSCov_t^{\Delta,k}$	1.258619	2.254377

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Ivana Filipovića 4 • 51000 Rijeka
tel +385 51 355 111
fax + 385 51 212 268
http://www.efri.uniri.hr
e-mail efri@efri.hr

$rRTSCov_t^{\Delta,k,\theta}$	1.295133	2.247213
Large jumps		
$rBPCov_t^{\Delta}$	1.598908	4.626791
$rCov_t^{\Delta}$	1.599028	4.626593
$rHYCov_t^{\Delta,\theta}$	1.599747	4.626648
$rThresholdCov_t^{k,h}$	1.599028	4.626593
$rTSCov_t^{\Delta,k}$	1.368619	3.427033
$rRTSCov_t^{\Delta,k,\theta}$	1.296134	3.422213

FIG 25. Robust two times scaled estimator correlation between Bitcoin and S&P500

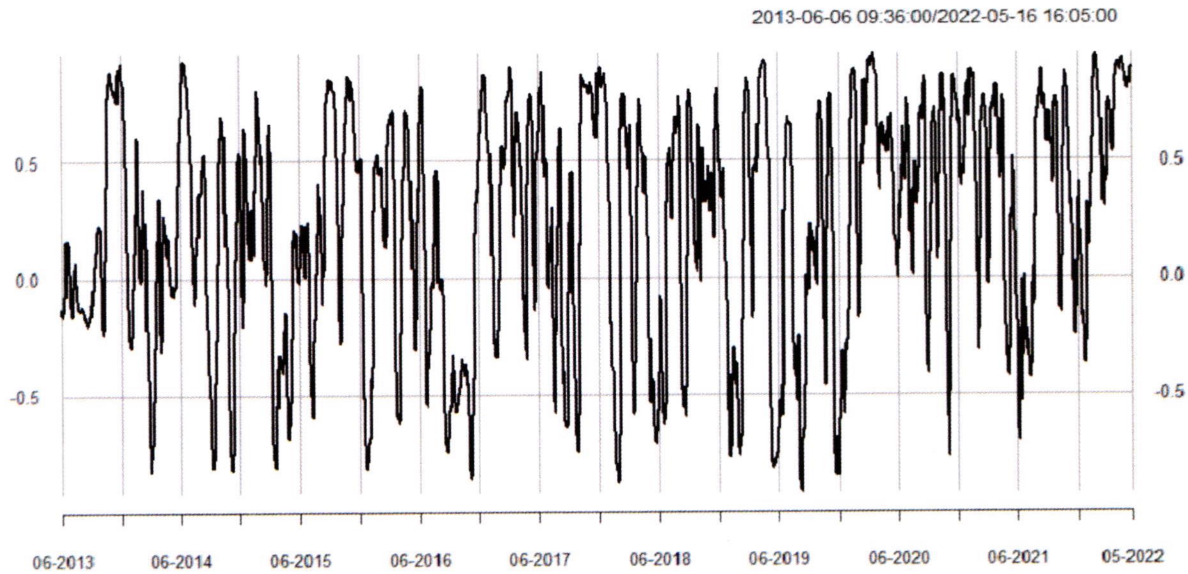

FIG 26. Robust two times scaled estimator correlation between Japanese Yen and S&P500

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Ivana Filipovića 4 • 51000 Rijeka
tel +385 51 355 111
fax + 385 51 212 268
http://www.efri.uniri.hr
e-mail efri@efri.hr

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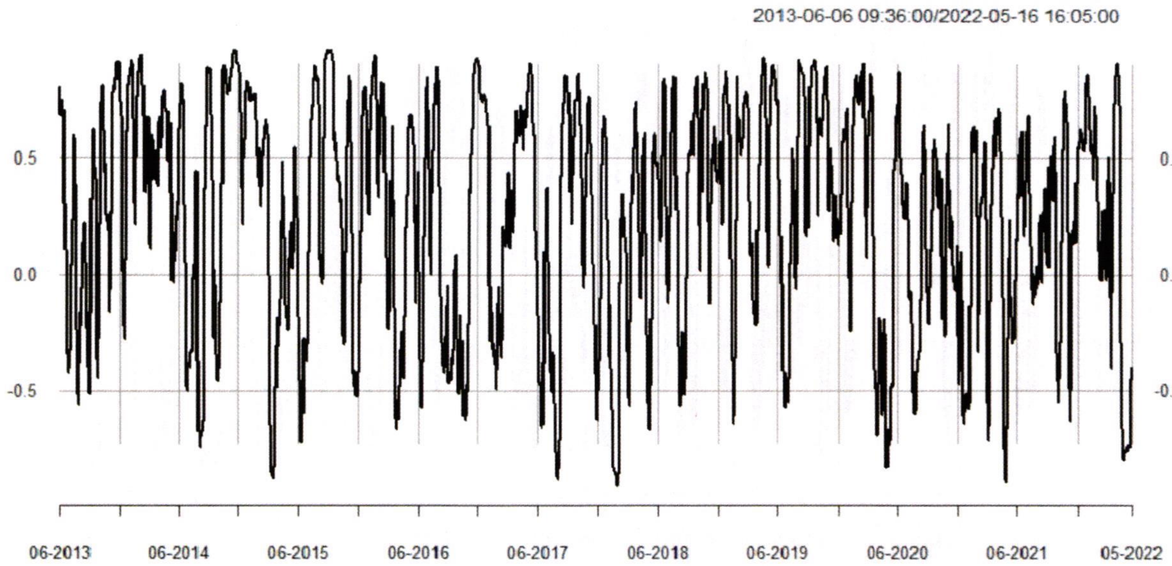

FIG 27. Robust two times scaled estimator correlation between U.S Dollar and S&P500

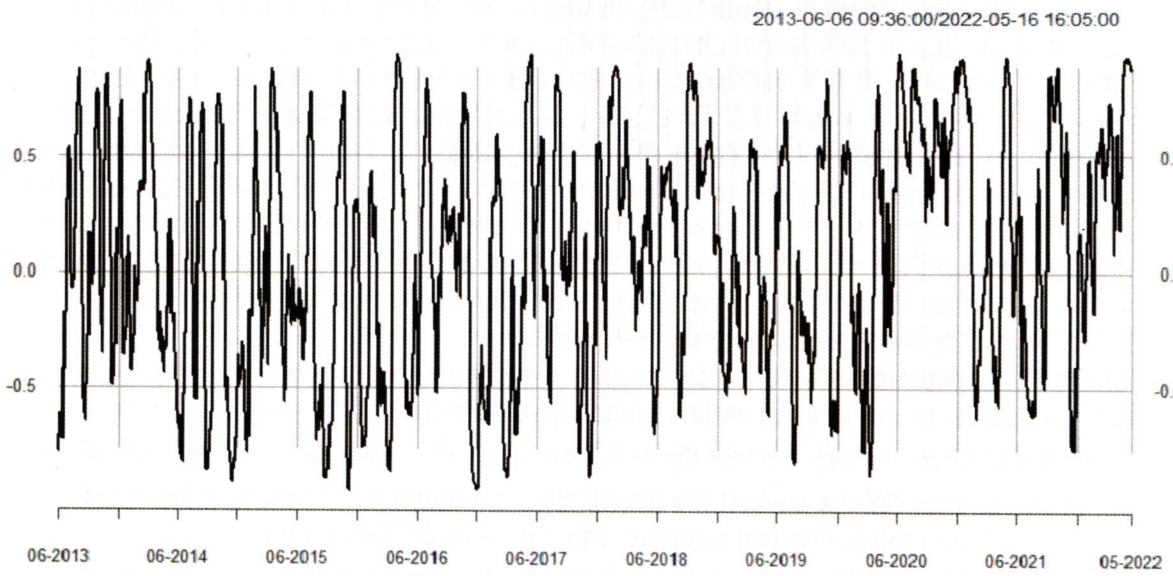

FIG 28. Robust two times scaled estimator correlation between Swiss Franc and S&P500

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Ivana Filipovića 4 • 51000 Rijeka
tel +385 51 355 111
fax + 385 51 212 268
http://www.efri.uniri.hr
e-mail efri@efri.hr

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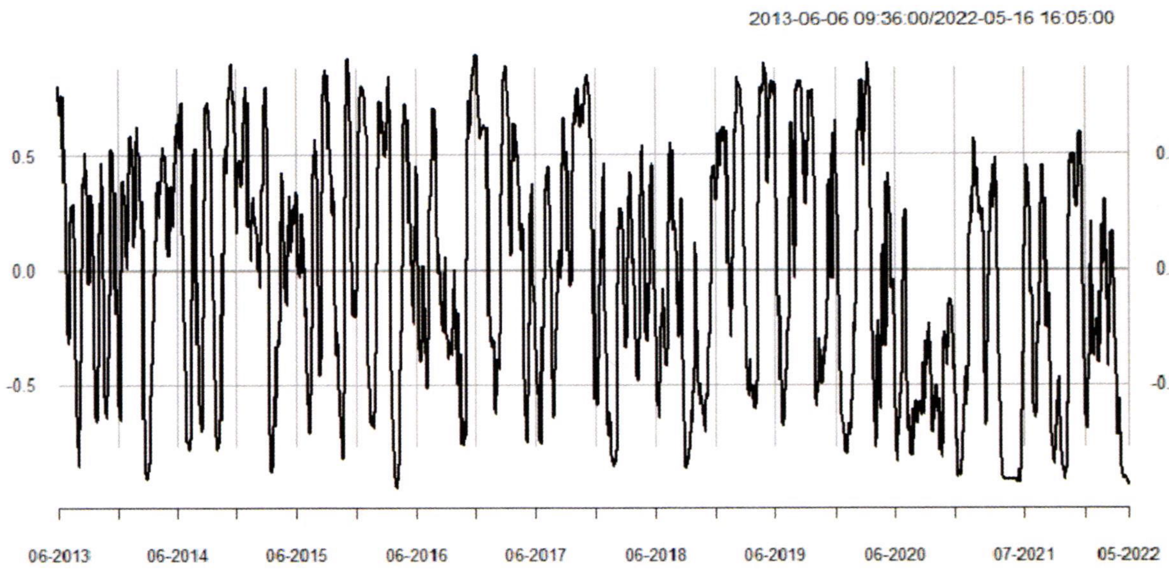

We estimated the proportion of negative correlation using Robust two times scaled estimator which would be an indicator which asset is "more" safe among the observed "safe haven" currencies. Negative correlation indicates safe haven characteristics as in Kaczmarek et al. (2022). The results have shown that Bitcoin had 49.35% estimated negative correlation with the general market in observed period and Japanese Yen had 29.79%. The U.S. Dollar had 46.06% and Swiss Franc had the highest estimated proportion of 50.46%. This estimated negative correlation clearly indicates that Bitcoin and Swiss Franc had the highest reverse movement from the general market in the observed period from June 2013 until May 2022. These results are a novelty in recent literature because of the use of benchmark realized covariance estimator – Robust two times scaled. In order to determine the safe haven characteristics from estimated negative correlation for each observed currency, we investigate further with a Chi-square test and one-tailed Z-test.

We continued with Chi-square test in order to compare the difference in population proportions between four groups i.e. proportions of estimated negative correlation of four different currencies with the general market (S&P500 index). We test the null hypothesis H_0: There is no significant difference between the observed proportions against the alternative hypothesis H_1: There is at least one proportion different from others. Our test resulted with a p-value 0.5998 and we therefore cannot reject the H_0 hypothesis on 5% significance level. In other words, there is no statistically significant difference among the proportions we were comparing. There is no statistical evidence to conclude that the proportions differ from each other. This shows us that all of the examined currencies have characteristics of safe haven currency as stated in the recent literature.

In order to elaborate more, we examined further and conducted a one-tailed Z-test. We were specifically interested in determining if the estimated proportion of negative correlation for each currency is significantly greater than the null hypothesis value. We performed the one-tailed Z-test for each observed currency on 5% significance level. The null hypothesis is $H_0: p = p_0$ where p_0 is the estimated proportion of negative correlation using Robust two times scaled estimator. The alternative hypothesis is $H_1: p > p_0$. The p-values are as follow respectively 0.4951 for Bitcoin, 0.9812 for Japanese Yen, 0.951 for U.S. Dollar and 0.00021 for Swiss Franc. Hence, we reject the null hypothesis

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Ivana Filipovića 4 • 51000 Rijeka
tel +385 51 355 111
fax + 385 51 212 268
http://www.efri.uniri.hr
e-mail efri@efri.hr

in case of Swiss Franc where we can conclude that the proportion is significantly greater than the specified tested value. Therefore we show that the Swiss Franc exhibits the best safe haven characteristics among competing alternative currencies. First contribution is that we used the high-frequency simulation study, which showed the superiority of Robust two times scaled covariance estimator. As a second contribution, we used Robust two times scaled covariance estimator for a comparative analysis to determine which currency has the best safe haven characteristics over periods that also includes times of market distress. The conclusions of this study provide valuable guidance for portfolio managers, policy makers, and retail investors.

This assessment can give clear prospective on the macro implications of cryptocurrencies whether the domestic institutions decide to regulate their status in order to guard sovereign currencies. The widespread adoption of cryptocurrencies could minimize the effectiveness of monetary policy in the long run but increase availability and usability for retail investors.

6. Conclusion

Since the beginning of the 21st century constructing optimal portfolios and hedging against market risks has become an ever more challenging task. We have witnessed a continuum of crises starting from dot-com bubble, 2008 global financial crisis, COVID crisis, energy crisis, European sovereign debt crisis and currently we are in the midst of the Russo-Ukrainian War and Chinese property sector crisis. The depth and continuity of crises is to a large extent due to greater connectivity between the markets and strong increase in cross-market and cross-assets correlations on regional or global scales. These characteristics are a sign of capital market becoming truly globalized and interconnected which carries with it a growing risk of setting a global stage for a perfect financial storm. Global interconnectedness decreases the opportunities to diversify ones' investments thus eliminating the possibility of effective hedging strategies. The effect of global financial contagion has not only practical implication on regular portfolio rotations and hedging strategies but also on the very foundations of modern portfolio theory and concept of international diversification of portfolios. This very unstable environment and ever-present fear of financial contagion effect is a key driver behind an increased interest in in safe haven assets. Safe haven assets are attractive as a research topic for their particular characteristic of providing a safe harbor during the times of financial stress.
In our research we focus on a special class of safe haven assets and that is safe haven currencies and the most famous digital asset/currency Bitcoin. In particular, we examine which potential safe haven currency is the best safe haven asset over the observation period based on the defined benchmark Robust two times scaled estimator of covolatility. Robust two times scaled covariance estimator proposed by Zhang (2011) has been repeatedly evaluated very positively in the recent literature, but no comprehensive comparison has been made between the realized covariance estimators to determine which one has the best accuracy. We used the high-frequency simulation study and analyzed the reduced relative bias and root mean squared error, which showed the superiority of Zhang's Robust two times scaled covariance estimator. Additionally, we used Robust two times scaled covariance estimator to determine which currency has the best safe haven characteristics over a long period of time from June 2013 to May 2022, which includes periods of serious market distress.
Our results show that cryptocurrencies, particularly Bitcoin, exhibit the characteristics of a safe haven currency compared to established safe haven currencies. Our analysis shows that Bitcoin and the Swiss Franc outperform the Japanese Yen and the U.S. Dollar in terms of safe haven characteristics, as they have a higher estimated negative correlation to the general market. The best performing safe haven currency is the Swiss Franc, using the estimated correlation with the benchmark Robust two

SVEUČILIŠTE U RIJECI
EKONOMSKI FAKULTET

Ivana Filipovića 4 • 51000 Rijeka
tel +385 51 355 111
fax + 385 51 212 268
http://www.efri.uniri.hr
e-mail efri@efri.hr

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times scaled estimator. All research was done based on high-frequency data which help to determine, firstly, benchmark volatility estimator Robust two times scaled estimator and secondly, benchmark covolatility estimator. Additionally, we examined potential safe haven asset, cryptocurrencies portfolios in in order to determine investment possibilities as well in market distress. Our results show that five of the six portfolio strategies performed better if they included sectoral cryptocurrencies namely from financial, exchange and business services sectors. Also, the results of this research provide insight for retail investors on investment opportunities by introducing cryptocurrencies into their portfolios. Further research should investigate hedging portfolios with cryptocurrencies as opposed to fiat currencies.

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**Wroclaw University of Economics and Business, PROM Programme, 30.04.2022 – 31.05.2022
Result was a published research paper „Machine Learning for Environmental Life Cycle Costing“ in Procedia Computer Science, publisher Elsevier**

Popis svih objavljenih radova iz područja istraživanja doktorske disertacije (obavljeno/planirano istraživanje)

1. Arnerić, Josip, and Maria Čuljak. "Predictive accuracy of option pricing models considering high-frequency data." Ekonomski vjesnik/Econviews-Review of Contemporary Business, Entrepreneurship and Economic Issues 34.1 (2021).

2. Čuljak, Maria, Josip Arnerić, and Ante Žigman. "Is Jump Robust Two Times Scaled Estimator Superior among Realized Volatility Competitors?." Mathematics 10.12 (2022): 2124.

3. Čuljak, Maria, Bojan Tomić, and Saša Žiković. "Benefits of sectoral cryptocurrency portfolio optimization." Research in International Business and Finance 60 (2022): 101615.

4. Paper "Putting safe haven currencies to the test using benchmark realized covariance estimator" is finished (with coauthors: Josip Arnerić, Saša Žiković and Gazi Salah Uddin) and is in preparation for submitting in a journal.

Publikacija rezultata istraživanja doktorske disertacije sudjelovanjem na međunarodnoj konferenciji (institucija, naziv, obavljeno/planirano istraživanje)

IFSA Publishing, Blockchain and Cryptocurrency Conference (B2C' 2023), 18-20 October 2023, Corfu, Greece
Presented a paper „Putting safe haven currencies to the test by using robust two times scaled covariance estimator.

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Ivana Filipovića 4 • 51000 Rijeka
tel +385 51 355 111
fax + 385 51 212 268
http://www.efri.uniri.hr
e-mail efri@efri.hr

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Humboldt University, Berlin, Germany, 01.10.2020 – 01.10.2021, „High Dimensional Nonstationary Time Series" **Result was a paper published in journal Mathematics: Čuljak, Maria, Josip Arnerić, and Ante Žigman. "Is Jump Robust Two Times Scaled Estimator Superior among Realized Volatility Competitors?." Mathematics 10.12 (2022): 2124.**

U Rijeci, 01.02.202
(datum)

Student:

Mentor:

Komentor: