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ANALYSIS AND ROBUSTNESS OF THE RETURN DISTRIBUTION ESTIMATORS, VOLATILITY AND COVOLATILITY OF STOCK MARKETS BY USING HIGH FREQUENCY DATA

DOCTORAL THESIS

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ANALIZA PROCJENITELJA DISTRIBUCIJE PRINOSA, VOLATILNOSTI I KOVOLATILNOSTI DIONIČKIH TRŽIŠTA POMOĆU VISOKOFREKVENTNIH PODATAKA I NJIHOVA OTPORNOST

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I dedicate this dissertation to my family, sister Viva and parents Vera and Srećko.

SUMMARY

This doctoral dissertation deals with the field of financial econometrics. Four financial market phenomena are investigated: returns distribution, volatility, covolatility, and safe haven assets using time series analysis and high-frequency data on financial assets.

The main objective is re-examining the safe haven and hedge properties of two currency classes, distinguished by the regulation and centralization level. While the role of a long-term hedge and safe haven during crisis periods, such as COVID, has been explored in the literature for various assets, this dissertation fills a significant gap by utilizing intraday transaction data and addresses numerous issues related to integrated variance and covariance estimation.

By sampling the data so frequently we get more complete information on price movement and trading activity.

The four steps that accompanied the four published papers led to the achievement of the goal - the answer to the main hypothesis: The robust two times scaled estimator is superior to other considered volatility and covolatility estimators. This hypothesis leads to the identification of a safe haven currency in comparison with the general market movements.

We start by examining distribution of returns and finding a data driven benchmark of the "true" density function for major market indices in consideration. Following that, we investigate whether the Robust two times scaled estimator is superior among alternative estimators of volatility by utilizing high-frequency data on a 1 second time scale over a 7-year period. Next we examine potential safe haven asset, cryptocurrencies market by comparing sectoral cryptocurrency portfolios with the benchmark CRIX index. Finally, we determine the best covolatility estimator and identify Swiss Franc as the best performing safe haven currency compared to general market movements and confirm that Bitcoin has the characteristics of safe haven currency.

SAŽETAK

Tema doktorske disertacije je iz područja financijske ekonometrije. Istražuju se četiri fenomena financijskog tržišta: distribucija prinosa, volatilnost, kovolatilnost te imovina sigurnog utočišta pomoću analize vremenskih serija i visokofrekventnih podataka financijske imovine.

Glavni cilj je preispitivanje svojstava sigurnog utočišta i zaštite dviju vrsta valuta koje se razlikuju po razini regulacije i centralizacije. Dok je uloga dugoročne zaštite i sigurnog utočišta tijekom kriznih razdoblja, kao što je COVID, istražena u literaturi za različitu imovinu, ova disertacija popunjava značajnu prazninu korištenjem podataka o unutardnevnim transakcijama i bavi se brojnim pitanjima povezanima s integriranom varijancom i procjenom kovarijance.

Tako čestim uzorkovanjem podataka dobivamo potpunije informacije o kretanju cijena i trgovinskoj aktivnosti.

Četiri koraka koja su pratila četiri objavljena rada dovela su do ostvarenja cilja - odgovora na glavnu hipotezu: Robusni procjenitelj s dvije skale je superioran u odnosu na druge razmatrane procjenitelje volatilnosti i kovolatilnosti. Ta hipoteza vodi do identifikacije sigurne valute u usporedbi s tržišnim kretanjima.

Počinjemo ispitivanjem distribucije prinosa i pronalaženjem referentne vrijednosti "prave" funkcije gustoće za glavne tržišne indekse koji se razmatraju. Nakon toga, istražujemo je li Robustni procjenitelj s dvije skale superioran među alternativnim procjeniteljima volatilnosti korištenjem visokofrekventnih podataka na vremenskoj skali od 1 sekunde tijekom razdoblja od 7 godina. U trećem koraku ispitujemo potencijalno sigurno utočište, tržište kriptovaluta, uspoređujući sektorske portfelje kriptovaluta s referentnim indeksom CRIX. Na kraju, utvrđujemo najbolji procjenitelj kovolatilnosti i identificiramo švicarski franak kao najbolju sigurnu valutu s najboljim rezultatima u usporedbi s općim tržišnim kretanjima i potvrđujemo da Bitcoin ima karakteristike sigurne valute.

PROŠIRENI SAŽETAK

Tijekom posljednja dva desetljeća bili smo svjedoci niza tekućih kriza počevši od dot-com balona, globalne financijske krize iz 2008., COVID krize, energetske krize, krize europskog državnog duga, rusko-ukrajinskog rata i krize kineskog sektora nekretnina. Ključno obilježje tih kriza bilo je snažno povećanje korelacija između tržišta i imovine na regionalnoj ili globalnoj razini. Ova karakteristika postaje sveprisutna opasnost koja signalizira rastući učinak globalne financijske zaraze na međunarodnim tržištima. Jasna prisutnost regionalne ili globalne financijske zaraze nema samo praktične implikacije na dnevnu raspodjelu portfelja i strategije zaštite od rizika, već također utječe na načine financijskog upravljanja. Tijekom kriznih razdoblja svi tržišni sudionici skloni su traženju utočišta u valutama koje se smatraju sigurnim utočištima. Oni pretvaraju svoju lokalnu gotovinu u te valute kako bi sačuvali vrijednost. Percipirana sigurnost tih valuta dovodi do povećanja njihove vrijednosti, čak i ako predmetni događaji možda nisu imali značajan utjecaj na tu valutu. Osim američkog dolara, švicarskog franka i japanskog jena koji su već poznati u literaturi kao valute sigurnog utočišta, tj. pouzdane pohrane vrijednosti tijekom nesigurnih ekonomskih vremena, kriptovalute također imaju potencijal djelovati kao imovina utočišta zbog svoje decentralizirane prirode i nedostatka regulacija. O njima se raspravljalo kao o potencijalnom sigurnom utočištu, osobito posljednjih godina kada je njihova popularnost porasla. Kriptovalute nisu vezane ni za jednu državu ili organizaciju, što ih čini manje ranjivima na djelovanja vlada ili geopolitičkih prijetnji. Ovo istraživanje ispituje i zaključuje koja je potencijalna valuta sigurnog utočišta bolja imovina sigurnog utočišta tijekom razdoblja promatranja na temelju definirane referentne vrijednosti Robustnog dvostruko skaliranog procjenitelja kovolatilnosti. Metodologija za određivanje cilja oslanja se na kovolatilnost. Kako bismo došli do zaključaka, podijelili smo naše istraživanje u četiri dijela. Ova doktorska disertacija bavi se područjem financijske ekonometrije, posebno analizom vremenskih serija visokofrekventnih podataka financijske imovine. Podaci visoke frekvencije, promatrani u vrlo kratkim vremenskim intervalima, primjerice svake minute ili svake sekunde, daju potpunije informacije ne samo o kretanju cijena, već i o trgovinskim aktivnostima, što omogućuje bolje razumijevanje pojava na financijskim tržištima, kao što su npr. distribucija prinosa, volatilnost i kovolatilnost. Procjena upravo ovih pojava pomoću visokofrekventnih podataka je način na koji zaključujemo o glavnoj hipotezi ove disertacije. Počinjemo ispitivanjem distribucije prinosa i pronalaženjem referentne vrijednosti "prave" funkcije gustoće za glavne tržišne indekse koji se razmatraju. Nakon toga, istražujemo je li Robustni procjenitelj s dvije skale superioran među alternativnim procjeniteljima volatilnosti korištenjem visokofrekventnih podataka na vremenskoj skali od 1 sekunde tijekom razdoblja od 7 godina. Zatim ispitujemo potencijalno sigurno utočište, tržište kriptovaluta, uspoređujući sektorske portfelje kriptovaluta s referentnim indeksom tržišta kriptovaluta, CRIX. Naposljetku, utvrđujemo najbolji procjenitelj kovolatilnosti i identificiramo sigurne valute s najboljim rezultatima u usporedbi s općim tržišnim kretanjima. Nakon uvodnog dijela doktorske disertacije slijede poglavlja u kojima su prezentirani sami znanstveni radovi koji čine ovu doktorsku disertaciju i čine zaokruženu logičku cjelinu.

Poglavlje 1. Prediktivna točnost modela određivanja cijena opcija s obzirom na podatke visoke frekvencije.

U prvom dijelu poglavlja, visokofrekventni podaci koriste se za procjenu funkcija gustoće vjerojatnosti za odabrane datume dospijeća europskih put i call opcija s ciljem njihove usporedbe s impliciranim funkcijama gustoće, izvedenim ex-ante (prije datuma dospijeća). Cilj ovog dijela istraživanja je korištenje visokofrekventnih podataka u određivanju snage predviđanja modela za određivanje cijena opcija. Ovdje se koriste visokofrekventni podaci za pružanje referentne funkcije gustoće vjerojatnosti koja se koristi u svrhu usporedbe. Osim visokofrekventnih podataka koji se promatraju svake minute, podaci o put i call opcijama na burzovne indekse CAC (Cotation Assistée en Continu), AEX (Amsterdam Exchange index), MIB (Milano Indice di Borsa) i DAX (Deutscher Aktien index) razmatraju se u ovom dijelu istraživanja. Istraživanje se provodi u dvije faze. Prva faza uključuje procjenu impliciranih funkcija gustoće vjerojatnosti na datum dospijeća korištenjem podataka o opcijama. Druga faza bavi se usporedbom procijenjenih funkcija gustoće vjerojatnosti s referentnom funkcijom gustoće na temelju visokofrekventnih podataka, dobivenom metodom Kernel procjene. Korištenje metode procjene gustoće kernela na promatranim visokofrekventnim podacima u stvarnom vremenu daje aplikativni doprinos i time veliku prednost u odnosu na druge studije koje se uglavnom oslanjaju na podatke simulacije. Korišteni modeli su: Shimkov model, Mixture Log-Normal model i Edgeworthov model ekspanzije. Glavni cilj istraživanja je procijeniti njihovu prediktivnu točnost i odabrati najprikladniji model. Promatrani burzovni indeksi su CAC, AEX, MIB i DAX, odnosno francuski, nizozemski, talijanski i njemački tržišni indeks. Financijski instrumenti koji se koriste su call i put opcije na glavne indekse navedenih financijskih tržišta na kombinacije datuma trgovanja i datuma dospijeća opcija u 2018. godini.

Procjenjujemo implicirane funkcije gustoće vjerojatnosti na datume dospijeća Shimkovog modela, Mixture Log-Normal modela i Edgeworthove ekspanzije. Pružamo rezultate usporedbe između procijenjenih funkcija gustoće vjerojatnosti, dobivenih trima modelima cijena opcija, i referentnih vrijednosti "pravih" funkcija gustoće vjerojatnosti, dobivenih procjenom Kernela korištenjem visokofrekventnih podataka. Za svaki datum dospijeća prikazana je grafička i analitička usporedba. U našem istraživanju implementiramo metode usporedbe izvan uzorka i utvrđujemo prediktivnu točnost. Korištena su dva testa, Diebold-Mariano test i Kolmogorov-Smirnov test. Rezultati su dostupni za sve kombinacije odabranih datuma trgovanja i isteka. Iz grafičkih rezultata očito je da je procjena gustoće kernela dovoljno točna da se može koristiti kao referentna vrijednost za usporedbu izvan uzorka i da uglavnom Shimkov model najbolje odgovara "pravoj" gustoći. Ovaj rezultat je od velikog interesa za investicijsku industriju kako bi analitičari znali koji model određivanja cijena opcija koristiti pri procjeni tržišnih očekivanja.

U većini slučajeva odbacujemo nultu hipotezu Kolmogorov-Smirnovljevog testa da procijenjene gustoće vjerojatnosti potječu iz "prave" funkcije gustoće. Nulta hipoteza je odbijena na razini značajnosti od 5% u većini slučajeva osim za DAX indeks na datum trgovanja 23. ožujka 2018. i datum dospijeća 21. rujna 2018. To znači da su funkcija gustoće vjerojatnosti implicirana Shimkovim modelom i "prava" funkcija gustoće dobivena Kernelovim procjeniteljem iste.

Diebold-Mariano test se koristi za testiranje nulte hipoteze da promatrani modeli cijena koji imaju istu sposobnost predviđanja. U primjeru AEX dioničkog indeksa na datum dospijeća 17. kolovoza 2018. i datum trgovanja 22. lipnja 2018. odbačena je nulta hipoteza na razini značajnosti od 5%. U jednakom broju slučajeva pokazalo se da Mixture Log-Normal model, Shimkov model i Edgeworthov model ekspanzije imaju istu prognostičku točnost, tj. nismo odbacili nultu hipotezu na razini značajnosti od 5%. Važno je naglasiti da su u slučaju DAX tržišnog indeksa na datum trgovanja 22. lipnja 2018. i datum isteka 20. srpnja 2018. svi modeli imali istu prognostičku točnost. Iz perspektive da Kernel procjenitelj pruža referentnu funkciju gustoće vjerojatnosti, može se zaključiti da je model Shimko najbolji model izvan uzorka u usporedbi s "pravom" gustoćom. Štoviše, nulta hipoteza Kolmogorvo-Smirnovljevog testa odbačena je u većini slučajeva za sve tržišne indekse i sve kombinacije datuma trgovanja i isteka. Rezultati Diebold-Mariano testa nisu odbacili nultu hipotezu koja implicira da modeli imaju istu prediktivnu točnost. Prema grafičkim prikazima i Kolomogorov-Smirnov testu možemo zaključiti da Shimkov model najbolju prognostičku točnost.

Ovo istraživanje pridonosi postojećoj literaturi na dva načina: prvenstveno za pronalaženje referentne vrijednosti "prave" funkcije gustoće korištenjem visokofrekventnih podataka pomoću Kernel procjenitelja te određivanje točnosti predviđanja modela cijena opcija. Usporedba referentne funkcije gustoće s procijenjenim funkcijama vjerojatnosti daje primjenjive rezultate za tržišne sudionike i financijska regulatorna javna tijela.

Poglavlje 2. Je li Jump Robust Two Times Scaled Estimator superioran među konkurentima ostvarene volatilnosti?

U drugom dijelu disertacije procjenjuje se integrirana varijanca odabranih burzovnih indeksa korištenjem visokofrekventnih podataka. Radi se o varijanci prinosa financijske imovine, odnosno standardnoj devijaciji prinosa koja se poistovjećuje s pojmom volatilnosti. Istražujemo je li robusni procjenitelj s dvostrukom skalom superioran među alternativnim procjeniteljima integrirane varijance korištenjem visokofrekventnih podataka na vremenskoj skali od 1 sekunde tijekom razdoblja od 7 godina. Razmatraju se dvije skupine procjenitelja, onih koji su otporni na mikrostrukturni šum i procjenitelji otporni na cijenovne skokove. Rad procjenitelja testiran je na burzovnim indeksima razvijenih europskih zemalja. Analizirani indeksi su likvidni, njima se učestalo trguje i pokazuju intenzivnu unutardnevnu aktivnost. Optimalna učestalost uzorkovanja svakog procjenitelja određena je s obzirom na kompromis između njegove pristranosti i varijance te je pojedinačno prilagođena značajkama svakog burzovnog indeksa. Osim testa transformacije integrala vjerojatnosti (PIT) i Mincer-Zarnowitzeve regresije, ovisnost o gornjem repu Gumbelove kopule smatra se prikladnom mjerom za usporedbu procjenitelja. Nadmoćnost robusnog procjenitelja s dvostrukom skalom dokazana je za sva analizirana tržišta s obzirom na optimalnu učestalost uzorkovanja. U ovom radu razmatraju se visokofrekventni podaci razvijenih burzi Njemačke, Italije, Francuske i Velike Britanije na fekvenciji uzorkovanja od 1 sekunde. Kako bi se definirala optimalna frekvencija uzorkovanja koristi se srednje kvadratna pogreška (RMSE) predloženog procjenitelja.

Nakon izračuna realiziranih procjenitelja volatilnosti i $RTSRV_t^{\Delta,k,\theta}$provode se tri metode usporedbe kako bi se odredilo koji procjenitelj volatilnosti najbolje odgovara referentnoj vrijednosti . Ove tri metode usporedbe su Mincer-Zarnowitzeva regresija, PIT test i ovisnost o gornjem repu Gumbelove kopule. Mincer-Zarnowitzeva regresija temelji se na ukupnoj izvedbi realiziranih modela volatilnosti. Proveden je PIT test kao postupak prilagodbe gustoće kako bi se ispitalo kako se ispitani realizirani procjenitelji volatilnosti ponašaju u usporedbi s $RTSRV_t^{\Delta,k,\theta}$. Korištena je ovisnost gornjeg repa jer ispituje što se događa u ekstremnim vrijednostima.

$RTSRV_t^{\Delta,k,\theta}$ je definiran kao referentna vrijednost jer je potvrđeno da je $RTSRV_t^{\Delta,k,\theta}$ otporan na mikrostrukturne šumove, cjenovne skokove i nesinkrono trgovanje.

Svaka metoda usporedbe uključuje prilagođavanje sedam procjenitelja volatilnosti referentnoj vrijednosti ($RTSRV_t^{\Delta,k,\theta}$). Optimalna učestalost uzorkovanja odabrana je za svako europsko tržište, na temelju minimiziranja srednje kvadratne pogreške (RMSE). Za izračun realizirane volatilnosti predlaže se korištenje prinosa koji se uzorkuju što je češće moguće. Međutim, ako je frekvencija uzorkovanja veća, to dovodi do problema s pristranošću zbog mikrostrukturnog šuma. Potreban je kompromis između pristranosti i učinkovitosti pri određivanju učestalosti uzorkovanja. Mora se uspostaviti optimalna učestalost uzorkovanja za promatrano financijsko tržište kako bi se smanjila pristranost, ali kako bi procjenitelj volatilnosti i dalje ostao učinkovit. U prisustvu skokova također će biti pristranosti u praktičnim primjenama procjenitelja koji nisu otporni na cjenovne skokove. Dakle, to će imati učinak na učestalost uzorkovanja na koju također utječu tržišna struktura, likvidnost i mikrostrukturni šum. Optimalna učestalost uzorkovanja utvrđena je za svako europsko tržište. Za Njemačku je to 20 sekundi, za UK je 30 sekundi, za Italiju je 10 sekundi i za Francusku je 13 sekundi. Minimiziranjem RMSE u odnosu na broj poduzoraka, dobiva se optimalna učestalost uzorkovanja.

Rezultati PIT testa i Mincer Zarnowitz regresije za usporedbu realiziranih procjenitelja volatilnosti pokazuju da RV_t^{Δ}, $TSRV_t^{\Delta,k}$, $ARV_t^{\Delta,k}$i $HYRV_t^{\Delta}$slabije podnose stres i cjenovne skokove. Stoga rezultati ne daju jasan odgovor koji procjenitelj najbolje odgovara referentonj vrijednosti. U tom slučaju, ovisnost o gornjem repu je povoljna metoda za korištenje jer uzima u obzir ekstremne vrijednosti. Prvo je korištena Mincer-Zarnowitzeva regresija gdje je testirano koliko dobro procjenitelji volatilnosti odgovaraju $RTSRV_t^{\Delta,k,\theta}$.

Za četiri europska tržišta i sedam procjenitelja volatilnosti nulta hipoteza je odbačena na razini značajnosti od 5%. To je pokazalo kako se sedam promatranih procjenitelja volatilnosti ne podudara s $RTSRV_t^{\Delta,k,\theta}$. Test integralne transformacije vjerojatnosti (PIT) korišten je kako bi se provjerilo je li razlika između $RTSRV_t^{\Delta,k,\theta}$ i drugih konkurentskih procjenitelja volatilnosti ravnomjerno raspoređena. Za sva europska tržišta i sve procjenitelje volatilnosti nulta hipoteza je odbačena na razini značajnosti od 5%. Ovo istraživanje koristi koeficijent ovisnosti o gornjem repu, rezultat funkcije Gumbelove kopule. Kada je fokus interesa gornji rep distribucije, on se koristi u svrhu usporedbe. Iako Mincer-Zarnowitzeva regresija i PIT test nisu pokazali prednost određenog procjenitelja, koristi se ovisnost gornjeg repa. Rezultati pokazuju da za Italiju, Njemačku i Ujedinjeno Kraljevstvo $RTSRV_t^{\Delta,k,\theta}$ postoji najveća ovisnost o gornjem repu s $MedRV_t^{\Delta}$, $MinRV_t^{\Delta}$ i BPV_t^{Δ} procjeniteljima volatilnosti. Među svim konkurentskim procjeniteljima, samo su oni otporni na cjenovne skokove i proizveli gotovo slične procjene volatilnosti kao $RTSRV_t^{\Delta,k,\theta}$. U slučaju Francuske, $RTSRV_t^{\Delta,k,\theta}$ najbolje odgovara , $TSRV_t^{\Delta,k}$, BPV_t^{Δ} i $ARV_t^{\Delta,k}$ procjeniteljima $MedRV_t^{\Delta}$ volatilnosti. Budući da je $TSRV_t^{\Delta,k}$ otporan na mikrostrukturni šum, nije iznenađenje da su procjene jednako dobre kao $RTSRV_t^{\Delta,k,\theta}$.

Zbog neuvjerljivih rezultata Mincer-Zarnowitzevog i PIT testa, uvedena je ovisnost gornjeg repa jer ispituje događaje u repovima, tj. ekstremne vrijednosti. Rezultati su pokazali da je medijanizirani blok od tri prinosa ($MedRV_t^{\Delta}$) imao najsličnije rezultate referentnoj vrijednosti ($RTSRV_t^{\Delta,k,\theta}$) za Italiju, Njemačku i UK. Za Francusku ($TSRV_t^{\Delta,k}$) procjenitelj realizirane volatilnosti bio je najsličniji (približno jednak) referentnoj vrijednosti. Budući da je medijanizirani blok od tri povrata ($MedRV_t^{\Delta}$) robustan samo na cjenovne skokove, zaključujemo da su talijanska, njemačka i britanska financijska tržišta više kontaminirana cjenovnim skokovima nego mikrostrukturnim šumom na odabranim frekvencijama uzorkovanja. Francusko financijsko tržište više je kontaminirano mikrostrukturnim šumom nego cjenovnim skokovima. Ovo istraživanje doprinosi postojećoj literaturi na nekoliko načina. Glavni rezultat razmatra odabir optimalne frekvencije u korist referenog procjenitelja na svakom tržištu pojedinačno, istovremeno osiguravajući otpornost na cjenovne skokove. Ovo istraživanje pridonosi prethodnim istraživanjima jer postoji vrlo malo studija koje uzimaju u obzir izračune realiziranih procjenitelja volatilnosti na razvijenim europskim tržištima. Novost je i korištenje kombinacije triju testova: Mincer-Zarnowitz regresije, PIT testa i testa ovisnosti gornjeg repa unutar Gumbelove kopule gdje se rezultati daju za svako od promatranih razvijenih europskih tržišta. Rezultati su važni financijskim analitičarima i investitorima jer daju preporuku koji realizirani procjenitelj volatilnosti koristiti za promatrane tržišne indekse. Dodatni doprinos je i utvrđena optimalna učestalost uzorkovanja za svako od promatranih razvijenih europskih tržišta.

Poglavlje 3. Prednosti sektorske optimizacije portfelja kriptovaluta.

U trećem dijelu ove disertacije formalno će se identificirati i opisati prednosti optimizacije portfelja sektorske klasifikacije kriptovaluta i njegove izvedbe. Formirat će se šest ciljeva optimizacije: MinVar, MinCVaR, MaxSR, MaxSTARR, MaxUT i MaxMean. Dobivene portfelje uspoređujemo s prinosom CRIX indeksa (koji predstavlja kripto tržište) u istom razdoblju. Naši

rezultati pokazuju da je pet od šest portfeljnih strategija imalo bolje rezultate ako su uključivale sektorske kriptovalute, i to iz sektora financijskih, deviznih i poslovnih usluga.
Za potrebe ove studije upotrijebili smo javno dostupne dnevne podatke o cijenama za ukupno 65 kriptovaluta prikupljene sa web stranice Coinmarketcap - CMC platforme. Kako bi se testirala korisnost sektorske podjele kriptovaluta, postojeći portfelj koji se sastoji od 50 najboljih kriptovaluta prema tržišnoj kapitalizaciji uključuje dodatnih 15 kriptovaluta, 5 vodećih kriptovaluta prema svakom od tri vodeća sektora upotrebe prema tržišnoj kapitalizaciji: financijskih, deviznih i poslovnih usluga. Sektorske kriptovalute koje su ušle među prvih 50 prema tržišnoj kapitalizaciji isključene su i zamijenjene sljedećim upotrebnim tokenom prema veličini tržišne kapitalizacije u odgovarajućem sektoru. Formiramo višestruke portfelje s različitim ciljevima optimizacije minimiziranja rizika, maksimiziranja povrata i maksimiziranja omjera povrata i rizika. Naši ciljevi optimizacije su sljedeći: minimalna varijanca (MinVar), minimalni CVaR (MinCVaR), maksimiziranje oštrog omjera (MaxSR), maksimiziranje stabilnog omjera povrata prilagođenog repu (MaxSTARR), maksimiziranje funkcije korisnosti (MaxUT) i maksimiziranje srednjeg povrata (MaxMean). Kako bismo ispitali prednosti analiziranja tržišta kriptovaluta kroz sektorsku podjelu, provodimo istraživanje u dva koraka. Prvi korak je formiranje i testiranje performansi portfelja čije komponente čine prvih 50 kriptovaluta prema tržišnoj kapitalizaciji. U drugom koraku, dodatnih 15 sektorskih kriptovaluta uključeno je u postojeći skup podataka. Kako bi se postiglo uključivanje sektorskih kriptovaluta u portfelj, u drugom koraku stvara se linearno grupno ograničenje pri čemu se 20% ukupne alokacije portfelja mora rasporediti na sektorske kriptovalute prema ciljevima optimizacije. Optimizacija se provodi izvan uzorka (backtesting), s istim parametrima za svaki cilj optimizacije.

Dobivene empirijske rezultate prezentiramo i interpretiramo u dvije faze. U prvoj fazi, rezultati se pregledavaju i interpretiraju komparativnom metodom između modela raspodjele imovine prema početnom odabiru komponenti portfelja. Osim toga, uspjeh određene strategije ocjenjuje se provedbom mjera izvedbe koje uključuju CRIX indeks kao referentnu vrijednost za kripto tržište tijekom razdoblja promatranja. U drugoj fazi, rezultati modela raspodjele uspoređuju se i tumače između portfelja kako bi se utvrdile prednosti podjele i optimizacije kriptovaluta prema njihovim odgovarajućim sektorima. Pokazalo se da, u slučaju stagnacije tržišta kriptovaluta, svaki od promatranih portfelja u prosjeku ostvaruje veće prinose od prinosa tržišta. Očekivano, najveću prosječnu alfu ostvario je MaxMean portfelj. S druge strane, valja istaknuti da je najveći godišnji geometrijski prinos, kao i kumulativni prinos u ukupnom razdoblju, ostvario portfelj s ciljem optimizacije minimiziranja CVaR-a. Stoga zaključujemo da se očekuje da će najbolje vrijednosti mjera izvedbe biti povezane s MinCVaR portfeljem.

U usporedbi s CRIX indeksom, svi implementirani optimizacijski ciljevi ostvarili su veći kumulativni povrat u istom razdoblju promatranja.

Uključivanjem dodatnih 15 sektorskih kriptovaluta koje u početku ne bi bile odabrane kao sastavnica portfelja prema njihovoj tržišnoj kapitalizaciji, rezultati se razlikuju po svim mjerama. Značajna razlika između ostvarenog geometrijskog prinosa MaxMean-S portfelja i CRIX indeksa, u usporedbi sa standardnom devijacijom aktivne premije koja je iznimno niska zbog podjednake volatilnosti između promatranih ulaganja, također je utjecala na visoko pozitivan Information ratio. U drugom redu najbolje veličine svih performansi, osim mjera rizika, postignut je portfeljem koji maksimizira omjer povrata i rizika izražen kao CVaR.

Najmanju standardnu devijaciju ostvarila je strategija optimizacije MaxSR-S, pri čemu je standardna devijacija MinVar-S portfelja bila nešto viša. U usporedbi s CRIX indeksom, svi optimizacijski ciljevi ostvarili su veći kumulativni povrat u istom razdoblju promatranja. Također je vrijedno istaknuti da su samo dvije strategije MaxUT-S i MaxMean-S postigle veću standardnu devijaciju od CRIX indeksa. Osim toga, svi su portfelji tijekom istog razdoblja ostvarili su viši Sharpeov omjer od CRIX indeksa.

Portfelji s dodatnim kriptovalutama u prosjeku ostvaruju više povrata od portfelja bez sektorskih komponenti. Geometrijski povrat ima iste odnose. Samo MinVar-S portfelj ima niži povrat od povrata portfelja bez sektorske kriptovalute, čime se potvrđuje naš nalaz i logika da postoje prednosti u tretiranju tržišta kriptovaluta kroz sektorsku pripadnost. Što se tiče rizika, četiri strategije koje uključuju sektorske kriptovalute postigle su nižu standardnu devijaciju od portfelja bez njih. Međutim, viši rizik je nadoknađen višim ostvarenim povratom, što implicira veći Sharpeov omjer. Uključivanje sektorskih kriptovaluta također je dovelo do povećanja kumulativnog povrata za sve strategije osim MinVar-S portfelja. Primjenom strategije maksimiziranja povrata i razmatranjem sektorskih kriptovaluta kao komponenti portfelja, bilo je moguće postići kumulativni povrat veći za 105% tijekom promatranog razdoblja od iste strategije koja ne uzima u obzir sektorske kriptovalute. Značajan porast kumulativnog prinosa također je ostvario MaxSTARR-S portfelj od 0,91 ili 69% u usporedbi s MaxSTARR-om. Uključivanje dodatnih sektorskih kriptovaluta u postojeće portfelje doprinosi poboljšanju performansi portfelja u usporedbi s tržištem koje predstavlja CRIX indeks. S obzirom na sve navedeno, može se zaključiti da je pet od šest portfelja kreiranih prema različitim optimizacijskim ciljevima postiglo bolje rezultate ako promatraju kriptovalute kroz sektorsku klasifikaciju (financijske, devizne i poslovne usluge). Ovakvi rezultati značajno pridonose istraživanju mogućnosti ulaganja na tržištu kriptovaluta i sektorskoj segmentaciji tržišta kriptovaluta. Osim toga, pozitivni rezultati upućuju na još dva zapažanja iz kojih je potrebno izvesti određene zaključke. Ako se kriptovaluta Litecoin koristi isključivo kao sredstvo plaćanja, usporedba Litecoina s decentraliziranom računalnom platformom poput Ethereuma i jednako tretiranje te dvije imovine u kontekstu mogućnosti ulaganja jednostavno nije praktično ili čak nemoguće, a to potvrđuju i naši rezultati. U blockchain ekonomijama poput Ethereuma, tokeni kao vrsta kriptovalute mogu se stvoriti kako bi pružili drugu svrhu i korisnost u praktičnoj primjeni proizvoda i usluge. Ova situacija dovela je do znatno boljih rezultata portfelja kada su uključene sektorske kriptovalute. Na tržištu kriptovaluta početak 2020. i 2019. obilježen je porastom vrijednosti utilization tokena, što je u određenoj mjeri predstavljalo Decentralized Finance - DeFi. Nijedna od 15 dodatnih kriptovaluta (tokena) odabranih po sektoru nije bila među 50 najboljih prema CMC-u, tako da nisu uzete u obzir u prethodnim studijama. Prethodni istraživački radovi smatrali su kriptovalute homogenom imovinom i oslanjali su se isključivo na opći algoritam optimizacije tržišta pri odabiru komponenti portfelja. Takav pristup podrazumijeva konsenzus o veličini ravnotežnog očekivanog povrata odabranih kriptovaluta. No, ranije se pokazalo da takav povrat niti ne postoji zbog nepostojanja odgovarajućeg vrednovanja, odnosno intrinzične vrijednosti kriptovaluta. Naše zaključke podupiru portfelji s dodatnim sektorskim kriptovalutama. U skladu s dobivenim rezultatima, naši nalazi naglašavaju korisnost i nužnost promatranja tržišta kriptovaluta po sektorskoj pripadnosti s ciljem pronalaska potencijalno "podcijenjenih" kriptovaluta. Ako su komponente portfelja odabrane isključivo prema tržišnoj kapitalizaciji, to bi značilo da su te kriptovalute već postigle vrijednost koja ih čini potencijalnom komponentom portfelja. Mogućnost rasta cijene takve kriptovalute svakako je puno manja od mogućnosti rasta kriptovalute koja je po tržišnoj kapitalizaciji znatno niže rangirana. Sektorski gledano, pojavljuju se kriptovalute s puno nižom tržišnom kapitalizacijom te ih investitori mogu lakše uočiti. Gledajući ukupnu kapitalizaciju sektora, lakše je uočiti i identificirati trenutne trendove na tržištu kriptovaluta, kao što je bio trend rasta DeFi kriptovaluta u 2019.

Rezultati sugeriraju da portfelji u kojima je 20% alokacije dodijeljeno kriptovalutama niže tržišne kapitalizacije postižu veće vrijednosti u svim implementiranim mjerama izvedbe u pet od šest optimizacijskih strategija. Može se zaključiti da je tržište kriptovaluta poželjno i potrebno promatrati kroz njihovu vrstu ili korisnost, a takav pristup se može postići kategorizacijom kriptovaluta u njihove sektore. Potencijalni ulagači, a posebno upravitelji portfelja, ne bi trebali razmatrati kriptovalute samo na temelju njihove tržišne kapitalizacije. Kriptovalute imaju karakteristike i mogućnosti koje ih definiraju prema njihovoj nominalnoj namjeni. Sukladno tome, upravitelji portfelja se potiču da prilikom izgradnje portfelja razmatraju kriptovalute prema njihovim karakteristikama (vrsti i namjeni koju pružaju) kako bi eliminirali svoj podređeni položaj i doprinijeli uspješnosti portfelja na tržištu kriptovaluta.

Glavni cilj našeg istraživanja bio je identificirati i opisati konstrukciju portfelja kreiranog prema različitim ciljevima optimizacije. Sastavni dijelovi ovih portfelja su kriptovalute kao nova vrsta imovine i po prvi puta kategorizirane u sektore koji će biti uključeni kao potencijalni kandidati za izgradnju portfelja. Predstavljeni rezultati sugeriraju da 20% alokacije portfelja dodijeljenog prema sektorskoj klasifikaciji kriptovaluta, pruža priliku za postizanje boljih mjera performansi portfelja u pet od šest optimizacijskih strategija. U našem istraživanju zaključeno je da sektorska klasifikacija identificira kriptovalute s većim potencijalom tržišnog rasta, kao i povećanjem diversifikacije i smanjenjem rizika portfelja. Ovo je novost i obogaćuje noviju literaturu o brzo rastućoj temi kriptovaluta. Vrijedan je za investitore i regulatore, ali posebno za upravitelje portfelja, tijekom praktične provedbe našeg istraživanja koje sugerira razmatranje kriptovaluta kao potencijalnih sastavnih dijelova portfelja kroz njihovu vrstu i vrijednost.

Poglavlje 4. Testiranje valuta sigurnih utočišta korištenjem benchmark realiziranog procjenitelja kovarijance.

U četvrtom i posljednjem poglavlju disertacije u kojem su prezentirani znanstveni radovi ispitali smo tri FX valute (američki dolar, švicarski franak i japanski jen) i Bitcoin u vremenskom periodu od 1 minute od lipnja 2013. do svibnja 2022. Studija je provedena u dvije faze. Prvo, korišteni je simulacija visokofrekventnih podataka kako bi se odredilo koji realizirani procjenitelj kovarijance ima najbolju izvedbu. Drugo, najbolji procjenitelj kovarijance, robusni procjenitelj s dvostrukom skalom, primjenjuje se na stvarne podatke visoke frekvencije, uspoređujući promatrane valute s tržištem dionica. Ovo istraživanje stoga daje dva doprinosa literaturi. Prvo, utvrđuje najbolji procjenitelj kovarijance i drugo, utvrđuje sigurne valute s najboljim rezultatima u usporedbi s općim tržišnim kretanjima. U trenutnoj literaturi ne postoji strogi konsenzus o superiornosti realiziranog procjenitelja kovarijance, pa će rezultati ovog istraživanja dati važan doprinos jer će sugerirati koji su procjenitelji prikladni za svako analizirano tržište i kako adresirati problem mikrostrukturnog šuma, cjenovnih skokova i asinkronih podataka, koji su usko povezani s odabirom optimalnih frekvencija uzorkovanja. Ne postoji konsenzus o tome koja je imovina bolje sigurno utočište, posebno u pogledu kriptovaluta kao što je Bitcoin. Odgovor ćemo dati u ovom istraživanju. Jedan od glavnih ciljeva ove studije je odrediti najbolji procjenitelj kovarijance za sinkronizirane i nesinkronizirane podatke visoke frekvencije. Prema rezultatima simulacijske studije, najbolji procjenitelj kovarijance koristi se za stvarni skup podataka visoke frekvencije.

Specifični ciljevi ove studije su:

a. Za ispitivanje procjenitelja kovarijance ($rBPCov_t^{\Delta}$, $rCov_t^{\Delta}$, $rHYCov_t^{\Delta,\theta}$, $rThresholdCov_t^{k,h}$, $rTSCov_t^{\Delta,k}$ i $rRTSCov_t^{\Delta,k,\theta}$) za sinkronizirane i nesinkronizirane visokofrekventne podatke.
b. Za usporedbu performansi procjenitelja kovarijancije na temelju mjera prilagodbe, tj. relativne pristranosti i srednje kvadratne pogreške (RMSE) za obje simulacije sinkroniziranih i nesinkroniziranih visokofrekventnih podataka.
c. Ispitati utjecaj skokova na model na svih šest procjenitelja kovarijancije za sinkronizirane i nesinkronizirane visokofrekventne podatke.

d. Koristeći dobivene rezultate o procjeniteljima kovarijance na emprijskim visokofrekventnim podacima, istražujemo koja je vrsta imovine bolje sigurno utočište. Negativna korelacija s općim tržištem ukazuje na karakteristike sigurnog utočišta.

Prvi cilj ovog rada je istražiti i odrediti najučinkovitiji procjenitelj kovarijance na temelju visokofrekventnih podataka. Naše istraživanje predstavlja dvofazni pristup za procjenu točnosti svakog procjenitelja. Prvo, koristimo simulaciju kako bismo pružili sveobuhvatnu procjenu točnosti svakog procjenitelja. Ovaj pristup osigurava da rezultati nisu pristrani i pouzdani. Drugo, koristimo zaključke izvedene iz rezultata simulacije za analizu visokofrekventnih na emprijskim podacima. Kako bismo osigurali točnost naših simulacija, slijedimo metodologiju postojećih istraživanja. Simuliramo za razdoblje od $S = 100$dana s frekvencijom od 1 sekunde. Dizajn simulacije omogućuje nam da usporedimo izvedbu svakog procjenitelja u različitim scenarijima i da identificiramo potencijalne pristranosti ili ograničenja u njihovoj točnosti. Time su rezultati našeg istraživanja robusni i pouzdani.

Šest realiziranih procjenitelja kovarijance ispitano je u ovom istraživačkom radu kako bi se odredilo koji se može koristiti kao referentna vrijednost. Prag kovarijance ($rThresholdCov^{k,\text{h}}{}_t$)procjenitelj uključuje univarijatna pravila za otkrivanje skokova kako bi ublažio utjecaj skokova na procjenu kovarijance. ($rCov_t^{\Delta}$) procjenitelj je robustan na mikrostrukturni šum ovisno o frekvenciji uzorkovanja. Skokovi cijena kontaminiraju procjene s $rCov_t^{\Delta}$. Stoga su uvedene modifikacije, kao što je Realized bipower procjenitelj ($rBPCov_t^{\Delta}$). Prednost $rBPCov_t^{\Delta}$je u tome što pruža dosljednu procjenu stvarne kovarijance, čak i u prisustvu cjenovnih skokova. Otporan je na cjenovne skokove, ali još uvijek na njega utječu mikrostrukturni šum i nelinearne ovisnosti između povrata imovine. Kako bi se izbjegao nedostatak podataka, ali i dalje ostao nepristran i asimptotski dosljedan procjenitelj, uveden je procjenitelj dvostruke kovarijance ($rTSCov_t^{\Delta,k}$). Stoga je otporan na mikrostrukturni šum i uključivanjem faktora skaliranja u procjenama, pristranost se može smanjiti. Budući da nije otporan na cjenovne skokove, uvedena je modifikacija. $rTSCov_t^{\Delta,k}$Robusni procjenitelj dvostruke kovarijance ($rRTSCov_t^{\Delta,k,\theta}$) robusna je verzija procjenitelja dvostruke kovarijance. Posljednja realizirana procjena kovarijance za usporedbu je Hayashi-Yoshida procjena kovarijance ($rHYCov_t^{\Delta,\theta}$) koju su predstavili Hayashi i Yoshida u 2005. godini. U prvom dijelu ovog istraživanja analiziraju se realizirani procjenitelji kovarijance na temelju simuliranih visokofrekventnih podataka. Učestalost simuliranih podataka je 1 sekunda za $S = 100$dana. Najprije se sinkronizirani visokofrekventni podaci koriste za analizu točnosti procjenitelja. Sinkronizirano znači da su podaci korišteni odmah nakon simulacije bez prilagođavanja učestalosti između dvije vremenske serije X_1i X_2korištenjem zadane frekvencije od 1 sekunde između promatranja. Relativna pristranost i srednja kvadratna pogreška izračunavaju se za svaki procjenitelj kako bi se izmjerila točnost kako se koristi u recentnoj literaturi. Najmanju relativnu pristranost imao je robusni procjenitelj s dvostrukom skalom ($rRTSCov_t^{\Delta,k,\theta}$) u slučaju kada u simuliranim podacima nije bilo skokova. U prisustvu skokova, vrijednosti relativne pristranosti se povećavaju pokazujući da se točnost procjenitelja smanjuje. Kada u simuliranim visokofrekventnim podacima nema skokova, rezultati pokazuju da je pristranost manja s procjeniteljima $rBPCov_t^{\Delta}$, $rCov_t^{\Delta}$, $rHYCov_t^{\Delta,\theta}$i $rThresholdCov_t^{k,h}$. Robustan dvostruki procjenitelj ($rRTSCov_t^{\Delta,k,\theta}$) koji je robustan na skokove cijena nije pokazao puno povećanja pristranosti bez obzira na intenzitet skokova u simuliranim podacima.

Na relativnu pristranost utječu skokovi, budući da se procjene relativne pristranosti povećavaju s povećanjem ili uključivanjem skokova za obje simulacije sinkroniziranih i nesinkroniziranih visokofrekventnih podataka. Nesinkronost visokofrekventnih podataka koju razmatramo su dvije neovisne sheme uzorkovanja Poissonovog procesa za generiranje serije stvarnih opažanja. Nema dokaza da uključivanje skokova poboljšava procjenitelje kovarijance u smislu snižavanja RMSE-a, kao što se vidi u sinkroniziranim i nesinkroniziranim visokofrekventnim podacima, RMSE procjenitelja kovarijance se povećava kako su skokovi uključeni u model. Analizirali smo šest procjenitelja kovarijance koji su $rBPCov_t^{\Delta}$, $rCov_t^{\Delta}$, $rHYCov_t^{\Delta,\theta}$, $rThresholdCov_t^{k,h}$, $rTSCov_t^{\Delta,k}$ i $rRTSCov_t^{\Delta,k,\theta}$ za sinkronizirane i nesinkronizirane visokofrekventne podatke. Naši rezultati simulacije nesinkroniziranih visokofrekventnih podataka pokazuju da $rRTSCov_t^{\Delta,k,\theta}$ je to najbolji procjenitelj kovarijance budući da ima najmanju relativnu pristranost za sve slučajeve (s ili bez skokova).

Dok za rezultate simulacije sinkroniziranih visokofrekventnih podataka, $rRTSCov_t^{\Delta,k,\theta}$ ima najmanju relativnu pristranost u usporedbi s drugim procjeniteljima kovarijance sa ili bez skokova. Ovaj rezultat podupire našu hipotezu budući da je pridruženi RMSE $rRTSCov_t^{\Delta,k,\theta}$ manji u usporedbi s RMSE drugih procjenitelja kovarijance jer su skokovi uključeni u model. Ovi rezultati pokazuju da $rRTSCov_t^{\Delta,k,\theta}$ nadmoćan u odnosu na druge procjenitelje kovarijance te ga nadalje koristimo kao refrentnu vrijednost.

Procijenili smo udio negativne korelacije korištenjem Robustnog dva puta skaliranog procjenitelja koji je pokazatelj koja je imovina "više " sigurna među promatranim valutama "sigurnog utočišta". Negativna korelacija ukazuje na karakteristike sigurnog utočišta kao što je pokazano u recentnoj literaturi. Rezultati su pokazali da je Bitcoin imao 49,35% procijenjene negativne korelacije s općim tržištem u promatranom razdoblju, a japanski jen 29,79%. Američki dolar je imao 46,06%, a švicarski franak je imao najveći procijenjeni udio od 50,46%. Ova procijenjena negativna korelacija jasno pokazuje da su Bitcoin i švicarski franak imali najznačajnije suprotno kretanje u odnosu na opće tržište u promatranom razdoblju od lipnja 2013. do svibnja 2022. Ovi su rezultati novost u novijoj literaturi zbog upotrebe referentnog realiziranog procjenitelja kovarijance. Kako bismo odredili karakteristike sigurnog utočišta iz procijenjene negativne korelacije za svaku promatranu valutu, dodatno istražujemo pomoću hi-kvadrat testa i jednostranog Z-testa. Nema statističkih dokaza koji bi mogli zaključiti da se omjeri međusobno razlikuju. To nam pokazuje da sve ispitane valute imaju karakteristike valute sigurnog utočišta. Nadalje, posebno smo bili zainteresirani za utvrđivanje je li procijenjeni udio negativne korelacije za svaku valutu značajno veći od vrijednosti nulte hipoteze. Proveli smo jednostrani Z-test za svaku promatranu valutu na razini značajnosti od 5%. Švicarski franak pokazuje najbolje karakteristike sigurnog utočišta među konkurentskim alternativnim valutama. Zaključci ove studije pružaju vrijedne smjernice za upravitelje portfelja, kreatore regulatornih politika te male ulagače.

Ova procjena može dati jasnu perspektivu o makro implikacijama kriptovaluta odluče li domaće institucije regulirati njihov status kako bi zaštitile suverene valute. Rašireno prihvaćanje kriptovaluta moglo bi smanjiti učinkovitost monetarne politike dugoročno, ali povećati dostupnost i upotrebljivost za male ulagače.

Zaključak doktorske disertacije

Od početka 21. stoljeća izgradnja optimalnih portfelja i zaštita od tržišnih rizika postali su sve izazovniji zadatak. Svjedoci smo niza geopolitičkih i tržišnih kriza. Dubina i kontinuitet kriza u velikoj je mjeri posljedica veće povezanosti između tržišta i snažnog porasta korelacija između tržišta i imovine na regionalnoj ili globalnoj razini. Ove karakteristike znak su da tržište kapitala postaje istinski globalizirano i međusobno povezano. Globalna međupovezanost smanjuje mogućnosti diversifikacije vlastitih ulaganja čime se eliminira mogućnost učinkovitih strategija zaštite. Ovo vrlo nestabilno okruženje i stalno prisutni strah od učinka financijske zaraze ključni su pokretač povećanog interesa za imovinu sigurnog utočišta. Imovina sigurnog utočišta privlačna je kao tema istraživanja zbog svoje posebne karakteristike pružanja sigurne luke u vrijeme financijskih kriza.

U našem istraživanju usredotočeni smo na posebnu klasu imovine sigurnog utočišta, a to su valute sigurnog utočišta i najpoznatija digitalna imovina/valuta Bitcoin. Konkretno, ispitujemo koja je potencijalna valuta sigurnog utočišta najbolja imovina sigurnog utočišta tijekom razdoblja promatranja na temelju definirane referentne vrijednosti Robustnog dvostruko skaliranog procjenitelja kovolatilnosti. Robustan dvostruki procjenitelj kovarijance koji je predložio Zhang u 2011. godini, je opetovano vrlo pozitivno ocijenjen u novijoj literaturi, ali nije napravljena sveobuhvatna usporedba između realiziranih procjenitelja kovarijance da bi se odredilo koji ima najbolju točnost. Upotrijebili smo studiju visokofrekventne simulacije i analizirali smanjenu relativnu pristranost i RMSE, što je pokazalo superiornost Zhangovog robusnog procjenitelja kovarijance. Osim toga, upotrijebili smo referentni procjenitelj kako bismo odredili koja valuta ima najbolje karakteristike sigurnog utočišta tijekom dugog vremenskog razdoblja od lipnja 2013. do svibnja 2022., što uključuje razdoblja ozbiljnih tržišnih kriza.

Naši rezultati pokazuju da kriptovalute, posebice Bitcoin, pokazuju karakteristike valute sigurnog utočišta u usporedbi s uspostavljenim valutama sigurnog utočišta. Naša analiza pokazuje da Bitcoin i švicarski franak nadmašuju japanski jen i američki dolar u pogledu karakteristika sigurnog utočišta, budući da imaju višu procijenjenu negativnu korelaciju s općim tržištem. Valuta sigurnog utočišta s najboljom izvedbom je švicarski franak, koristeći procijenjenu korelaciju s referentnim procjeniteljem. Sva su istraživanja provedena na temelju visokofrekventnih podataka koji pomažu u određivanju, prvo, referentne vrijednosti procjenitelja volatilnosti Robustnog dvostruko skaliranog procjenitelja i drugo, referentne vrijednosti procjenitelja kovolatilnosti . Osim toga, ispitali smo potencijalnu imovinu sigurnog utočišta, portfelje kriptovaluta kako bismo odredili mogućnosti ulaganja, i za vrijeme tržišnih kriza. Naši rezultati pokazuju da je pet od šest portfeljnih strategija imalo bolje rezultate ako su uključivale sektorske kriptovalute, i to iz sektora financijskih, deviznih i poslovnih usluga. Također, rezultati ovog istraživanja pružaju uvid malim ulagačima u mogućnosti ulaganja uvođenjem kriptovaluta u njihove portfelje. Daljnja istraživanja trebala bi istražiti zaštitu portfelja s kriptovalutama za razliku od fiat valuta.

KEYWORDS

Keywords: High-frequency observations, Realized Volatility, Microstructure Noise, Price Jumps, Sampling Frequency, Cryptocurrency, Portfolio Optimization, Sectoral Classification, Investments, Safe haven, Realized Covariance

Ključne riječi: Visokofrekventna promatranja, Realizirana volatilnost, Mikrostrukturni šum, Cijenovni skokovi, Učestalost uzorkovanja, Kriptovaluta, Optimizacija portfelja, Sektorska klasifikacija, Ulaganja, Sigurno utočište, Realizirana kovarijanca

CONTENTS

1. INTRODUCTION

This doctoral dissertation is situated within the domain of financial econometrics, with a particular focus on the analysis of high-frequency time series data pertaining to financial assets. High-frequency data, recorded at ultra-short intervals such as every second or minute, offer an unprecedented granularity of insight into financial markets. Such data not only capture detailed price dynamics, but also reflect the underlying trading activity and microstructure noise, enabling more accurate modelling and interpretation of complex market phenomena. These include, but are not limited to, the empirical characteristics of return distributions, the temporal dynamics of volatility, and the co-movements of asset returns, known as co-volatility. The overarching aim of this dissertation is to enhance the econometric modelling of these phenomena by leveraging the richness of high-frequency data. To this end, the work begins with an empirical investigation of the return distribution of major financial market indices. The goal of this initial phase is to establish a benchmark or reference density function that accurately reflects the empirical characteristics of asset returns when measured at high temporal resolution. This benchmark serves as a foundational element for subsequent analysis. Building on this, the study evaluates the performance of volatility estimators in the context of high-frequency data. Specifically, the robustness and efficiency of the Robust Two-Scale Realized Volatility Estimator (RTSE) are assessed in comparison to a set of alternative estimators. The empirical analysis utilizes a comprehensive dataset sampled at 1-second intervals over a 7-year horizon, allowing for a thorough and statistically robust evaluation of estimator performance under realistic market conditions. The dissertation then extends its scope to explore the cryptocurrency market, which has emerged as a potential safe haven and alternative asset class in times of economic uncertainty. Using a portfolio-based approach, the analysis compares sectoral cryptocurrency portfolios with a well-established benchmark index, the CRIX (CRyptocurrency IndeX), to assess their relative performance and potential diversification benefits. This investigation sheds light on the evolving role of digital assets in modern financial systems and their implications for portfolio construction and risk management. In the final component of the dissertation, attention is turned to the estimation of co-volatility structures. Identifying accurate and reliable estimators of co-volatility is critical for understanding systemic risk and for constructing robust hedging strategies. The study evaluates a suite of co-volatility estimators and examines their performance in capturing the interdependencies between financial assets. Furthermore, the analysis identifies specific currencies, including those within the digital asset space, that exhibit safe haven properties by demonstrating resilience or inverse correlation with broader market downturns.

The first empirical investigation undertaken in this dissertation seeks to identify the option pricing model that most accurately predicts the implied probability density function (PDF) of an asset's terminal value, as inferred from market prices of options prior to maturity (ex-ante). This prediction is then rigorously evaluated by comparing it with the realized PDF (ex-post), which is derived from high-frequency data observed at the actual maturity date. By conducting this comparison, the study assesses the forecasting accuracy and overall predictive power of various option pricing models, namely, parametric, semi-parametric, and non-parametric approaches. The ultimate objective is to determine which model provides the most reliable estimation of future market behavior, thereby supporting improved pricing, risk assessment, and hedging strategies. The central hypothesis guiding this analysis (H1) posits that the non-parametric option pricing model has a higher predictive power in estimating the future probability density function compared to the parametric and semi-parametric model. This hypothesis is grounded in the expectation that non-parametric methods, by avoiding restrictive assumptions about the underlying distribution, can more flexibly and accurately capture the complex dynamics embedded in option prices and the associated implied information about future market states. Taken together, the findings of this dissertation contribute to the field of financial econometrics by advancing methodologies for high-frequency data analysis, offering new insights into asset behavior under varying market conditions, and enhancing our understanding of volatility dynamics and safe haven asset identification in both traditional and emerging markets.

The second core empirical contribution of this dissertation focuses on the comparative evaluation of realized variance estimators, with the objective of identifying the estimator that most accurately approximates the unobservable integrated variance of asset returns. Recognizing the distortions introduced by market microstructure effects and discontinuities such as price jumps, the study critically assesses a range of estimators, with particular emphasis on the robust two-time-scale estimator (RTSE). This estimator is specifically designed to mitigate the influence of microstructural noise and to provide reliable estimates in the presence of abrupt price changes. Through a comprehensive empirical analysis, the predictive performance and robustness of the RTSE are benchmarked against two broad groups of alternative realized variance estimators. Furthermore, the analysis is applied across a selection of developed European equity markets to investigate the extent to which these markets are contaminated by price jumps and microstructural noise. Understanding these market-specific characteristics is essential for determining the conditions under which certain estimators outperform others. The guiding hypothesis for this part of the research (H2) asserts that the robust two-time-scale estimator, which is resistant to microstructural noise and price jumps, is superior to other estimators of realized variance. This hypothesis is tested empirically using high-frequency intraday data, enabling a detailed evaluation of estimator performance across varying market conditions and microstructure environments.

The third paper of this dissertation explores investment opportunities within the cryptocurrency market, with a particular focus on the role of sectoral portfolio optimization. In the first phase of the analysis, a comparative evaluation of various asset allocation models is conducted, whereby the outcomes are analyzed based on the initial composition of the cryptocurrency portfolios. This phase aims to identify how different selection strategies influence the performance and risk-return profiles of the resulting portfolios. In the second phase, the study advances to a cross-portfolio comparison, in which the performance of allocation models is assessed across sectorally structured cryptocurrency portfolios. This phase seeks to determine the extent to which categorizing and optimizing cryptocurrencies according to their underlying sectors (such as DeFi, infrastructure, or payments) enhances diversification benefits and improves overall investment efficiency. By addressing these two dimensions, the paper provides empirical evidence on the value of sector-based differentiation in crypto-asset management and contributes to the growing literature on systematic investment strategies in digital financial markets.

The last paper contributes to the final objective of this disertation. It contributes to the existing body of literature by employing a high-frequency simulation framework to evaluate the performance of covariance estimation techniques, with a particular focus on the robust realized two-scale covariance estimator (rRTSCov). The study provides empirical evidence supporting the superiority of rRTSCov in accurately capturing the co-movements of asset returns in the presence of market microstructure noise and asynchronous trading, conditions commonly observed in high-frequency financial data. Furthermore, the paper examines the role of various assets, both traditional currencies and cryptocurrencies, as potential safe haven instruments during periods of financial stress. Using Bitcoin (BTC) as the representative cryptocurrency, the findings reveal that BTC exhibits certain safe haven properties, albeit to a lesser extent than conventional fiat currencies. Among the traditional currencies analyzed, the Swiss franc (CHF) consistently demonstrated the strongest safe haven characteristics, outperforming all other currencies in the sample. These findings enhance our understanding of asset behavior under stress conditions and underscore the importance of robust covariance estimation in portfolio risk management and safe haven identification.

2. ELABORATION OF THE THESIS

Abstract

This thesis summary is divided into four papers, each organized into a chapter. Chapter 1: The first research paper, titled "Predictive Accuracy of Option Pricing Models Considering High-frequency Data" determines the benchmark of the "true" density function using high-frequency data within the Kernel estimator. It also assesses the predictive accuracy of the option pricing models. Chapter 2: The second research paper, titled "Is the robust two times scaled estimator superior among competing alternatives of realized volatility?" investigates whether the robust two-times scaled estimator is superior among alternative estimators of integrated variance using high-frequency data on a 1-second time scale over seven years. Chapter 3: The third research paper, "Benefits of Sectoral cryptocurrency portfolio optimization" identifies and describes the benefits of sectoral cryptocurrency classification portfolio optimization and its performance. Chapter 4: The fourth research paper, titled “Re-examining Safe Havens and Hedges through a Realized Covariance Lens" identifies the best covariance estimator and the best-performing safe-haven currencies compared to general market movements.

Keywords: High-frequency observations, Realized Volatility, Microstructure Noise, Price Jumps, Sampling Frequency, Cryptocurrency, Portfolio Optimization, Sectoral Classification, Investments, Safe haven, Realized Covariance

Chapter 1

Predictive Accuracy of Option Pricing Models Considering High-frequency Data

Arnerić, J., & Čuljak, M. (2021). Predictive accuracy of option pricing models considering high-frequency data. Ekonomski vjesnik/Econviews-Review of Contemporary Business, Entrepreneurship and Economic Issues, 34(1). pp. 14.

When modeling high-frequency data, the key questions are which model, machine learning algorithms (supervised or unsupervised learning) to use, is the data classified or not, is the data linearly separable or not? We need to answer all these questions before, then we think about the models to use for modeling in question. The accuracy of the model is key when making decisions about forecasting and predictability. For the case of option pricing models, model predictive accuracy needs to be incorporated with the entire probability density function of the underlying asset. We employ call and put European option prices for this study. We identify the model that predicts the entire probability density function (pdf) most accurately when compared to the ex-post "true" density given by high-frequency data at the expiration date. To achieve this, we first estimate several probability density functions using different option pricing models, considering data of major market indices with different maturities, these implied probability density functions are risk-neutral at the expiration date. Then the implied PDFs are compared against the "true" density obtained from the high-frequency data to examine which one gives the best fit out-of-sample. The "true" density function is unknown, but it can be estimated using high-frequency data adjusted for risk preferences. We find a data-driven benchmark of the "true" density function for major market indices in consideration. The "true" density function using high-frequency data within the Kernel estimator is used to determine the predictive accuracy of the option pricing models. For applications of our results, we compare the density function against estimated risk-neutral probability functions for market participants and public authorities, respectively.

We employ high-frequency data to construct a reference probability density function, which serves as a benchmark for assessing the predictive performance of option pricing models. The use of high-frequency data is justified by advancements in trading technologies and the ability to record nearly all transactions. The increasing prevalence of electronic trading in regulated markets and multilateral platforms is driven by algorithmic developments, faster order placements, and enhanced liquidity. These factors contribute to unprecedented levels of market activity at high frequency, resulting in richer, higher-quality information. Both academics and practitioners are drawn to intraday data, collected at very short intervals, as it captures the ever-evolving nature of market structures and trading dynamics. Our dataset also incorporates data from electronic trading platforms that have automated and accelerated trade execution and reporting, enabling investors to implement automated strategies and manage orders in real time. The put and call options data on the stock market indices CAC (Cotation Assistée en Continu), AEX (Amsterdam Exchange index), MIB (Milano Indice di Borsa), and DAX (Deutscher Aktien index) are considered for forecasting the future expectation, variance, and higher moments of financial assets. We first estimate the implied probability density functions at the expiration date using options data and then compare the estimated probability density functions against the reference density function based on high-frequency data, obtained by the Kernel estimation method. Like other studies that use simulated data, our study employs the Kernel density estimation method on observed high-frequency data in real-time which provides an applicative contribution. Comparing the reference probability density function with an estimated risk-neutral density function results in recommendations for academics and practitioners, particularly for financial analysts and market participants, as additional information on risk preferences can be found. According to Arnerić and Čuljak (2021), there are some issues regarding the high-frequency data, such as financial market illiquidity, and thus lack of data and doubts about sampling frequency selection since time intervals are required to be equidistant and non-overlapping.

We then evaluate the predictive accuracy of the used models from a class of non-parametric, parametric, and semi-parametric option pricing models: Shimko model, Mixture Log-Normal model, and Edgeworth expansion model. These models provide the best fit for the data and have the highest predictive accuracy simultaneously. For implied risk-neutral density estimation and the option pricing models (Mixture Log-Normal model (MLN), Edgeworth expansion model (EE) and Shimko model (SM)) employed in this paper and an overview of option pricing that reduce the limitations of Black and Scholes model, see Arnerić and Čuljak (2021).

High-frequency data exhibits distinct characteristics, one of which is microstructure noise (the deviation of observed prices from fundamental values, often caused by factors like bid-ask spreads, information asymmetry, discrete price movements, and order processing delays). This feature can lead to biased estimates of certain parameters. Another key aspect of high-frequency data is its discrete nature—observations are not captured continuously but instead occur at irregular time intervals or unevenly spaced points in time. High-frequency data are also characterized by non-normality, i.e. they show the property of fat-tails. These features make determining an appropriate distribution complex. However, finding a true, but unknown daily distribution based on intraday prices becomes possible using the Kernel density estimation method. The "true" probability density function can be obtained for each maturity date and then compared with ex-ante density functions derived from several option pricing models. With this idea, we compare implied risk-neutral probability density functions and the estimated probability density functions as benchmarks of the "true" densities. High-frequency data were obtained from the Thomas Reuters database for the same expiration dates, for which the kernel density method is used to estimate the "true" probability density functions. The observed stock indices are the French (CAC), Dutch (AEX,), Italian (MIB), and German (DAX) market index. We use a Gaussian kernel function, i.e. $K(x) = \phi(x)$, where ϕ is the standard normal probability density function. The kernel primarily estimates the probability density function at each data point and then sums all these densities to produce a final estimate. The choice of the kernel function has no significant effect on the final density estimation.

According to Rosenberg and Engle (2002) and Šestanović and Arnerić (2021), the choice of the kernel function does not affect the outcome, while it is more sensitive to the bandwidth. This research adjusts the kernel bandwidth parameter to obtain the best fit concerning the high-frequency data for each expiration date under consideration. Further, we estimate the implied probability density functions on the expiration dates of the Shimko model, Mixture Log-Normal model, and Edgeworth expansion. Financial instruments used to call and put options on the major indices of the listed financial markets on combinations of options trading dates and options expiration dates in 2018, see Table 1.1. From Table 1.1, we can see that options data are not available for all market indices at given trading dates. On the trading date of May 18, 2018, options data are available for all four indices AEX, CAC, DAX, and MIB with expiration on September 21, 2018 (maturity horizon approximately 4 months), but only for the DAX index with expiration on July 20, 2018 (maturity horizon one month).

For the results of this research, we implement out-of-sample comparison methods and determine predictive accuracy. We then compare the estimated probability density functions, obtained by the three option pricing models, and benchmarks of the “true” probability density functions, obtained by Kernel estimation using high-frequency data. A graphical and analytical comparison is presented at each maturity date. The study that is most similar to our research compares three parametric density functions obtained by a mixture of two log-normal (MLN), Black-Scholes-Merton (BSM), and generalized beta (GB2) according to Arnerić (2020). We use Mean square error (MSE) and absolute relative error (ARE) for pairwise comparison purposes only, neglecting the “true” probability density function that can be observed ex-post—Diebold-Mariano test (DM) tests which model has a lower MSE. Two tests were used here, the Diebold-Mariano test and the Kolmogorov-Smirnov test (Pauše, 1993).

The parametric models are usually overfitted making a wrong impression of how these models fit the data. Due to the unique characteristics of the proposed models, we consider them to be sensitive to different maturity dates. Additionally, because semi-parametric and non-parametric approaches do not explicitly form the risk-neutral probability density function and there is no assumption about the function itself, we focus on the Shimko model (SM), Mixture Log-Normal model (MLN), and Edgeworth expansion model (EE). For the prices of call and put options, the midpoints between bid and ask prices are taken. EURIBOR is a risk-free interest rate, depending on the forecast horizon. The forecast horizon varies from 1 month to 6 months see Table 1.1. The results for the AEX, CAC, DAX, and MIB index are presented graphically from Figure 1 to Figure 18 (Appendix) see Arnerić and Čuljak (2021), from these figures, it can be observed that Kernel density estimation is accurate enough to be used as a benchmark for comparative purposes out-of-sample and that mostly Shimko model fits the “true” density the best.

Table 1.1: Options trading dates and expiration dates concerning four stock market indices

The year 2018	Options expiration dates		
Options trading dates	July 20	August 17	September 21
March 23			ALEX, DAX, MIB
April 20			AEX, CAC
May 18	DAX		AEX, CAC, DAX, MIB
June 22	AEX, CAC, DAX, MIB	AEX, DAX	AEX, DAX

Source: Thomson Reuters

The figures from Figure 1 to Figure 18 (Appendix) see Arnerić and Čuljak (2021), present a graphical comparison of implied probability density functions using three different option pricing models (MLN, EE, SM) and Kernel estimated probability density function based on high-frequency data. It is observed that Kernel density estimation is accurate enough to be used as a benchmark for comparative purposes out-of-sample and that the Shimko model fits the "true" density the best. This observation applies to the investment industry since analysts can know which option pricing model to use when assessing market expectations.

Table 1.2: Comparison results from the Kolmogorov-Smirnov test and Diebold-Mariano test

Index / Expiration date / Trading date	Kolmogorov-Smirnov test			Diebold-Mariano test		
	TD-MLN	TD-SM	TD-EE	TD-MLN	TD-SM	TD-EE
ALEX						
August 17, 2018 June 22, 2018	0,26 (p<0,05)	0,54 (p<0,05)	0,67 (p<0,05)	-8,85 (p<0,05)	-7,51 (p<0,05)	-6,85 (p<0,05)
July 20, 2018 June 22, 2018	0,41 (p<0,05)	0,52 (p<0,05)	0,59 (p<0,05)	2,23 (p<0,05)	0,85 (p>0,05)	-2,21 (p<0,05)
September 21, 2018 March 23, 2018	0,38 (p<0,05)	0,61 (p<0,05)	0,72 (p<0,05)	-6,75 (p<0,05)	-7,81 (p<0,05)	-1,32 (p<0,05)
September 21, 2018 April 20, 2108	0,42 (p<0,05)	0,65 (p<0,05)	0,75 (p<0,05)	-5,74 (p<0,05)	-7,34 (p<0,05)	-4,24 (p<0,05)
September 21, 2018 May 18, 2018	0,37 (p<0,05)	0,59 (p<0,05)	0,65 (p<0,05)	-7,24 (p<0,05)	-8,21 (p<0,05)	-6,99 (p<0,05)
September 21, 2018 June 22, 2018	0,36 (p<0,05)	0,51 (p<0,05)	0,6 (p<0,05)	-5,41 (p<0,05)	-7,96 (p<0,05)	-7,38 (p<0,05)
CAC						

July 20, 2018	0,53	0,80	0,38	-4,06	-7,55	-0,87
June 22, 2018	(p<0,05)	(p<0,05)	(p<0,05)	(p<0,05)	(p<0,05)	(p<0,05)
September 21, 2018	0,54	0,47	0,54	-7,04	-8,62	-5,41
April 20, 2018	(p<0,05)	(p<0,05)	(p<0,05)	(p<0,05)	(p<0,05)	(p<0,05)
September 21, 2018	0,31	0,35	0,53	-7,99	-9,82	-8,58
May 18, 2018	(p<0,05)	(p<0,05)	(p<0,05)	(p<0,05)	(p<0,05)	(p<0,05)
August 17, 2018	0,35	0,47	0,53	-18,06	-10,94	-7,00
June 22, 2018	(p<0,05)	(p<0,05)	(p<0,05)	(p<0,05)	(p<0,05)	(p<0,05)
DAX						
July 20, 2018	0,37	0,55	0,38	-5,85	-1,45	-5,68
May 18, 2018	(p<0,05)	(p<0,05)	(p<0,05)	(p<0,05)	(p>0,05)	(p<0,05)
July 20, 2018	0,31	0,46	0,49	1,94	1,34	-0,76
June 22, 2018	(p<0,05)	(p<0,05)	(p<0,05)	(p>0,05)	(p>0,05)	(p<0,05)
September 21, 2018	0,34	0,15	0,13	0,34	-5,18	-7,70
March 23, 2018	(p<0,05)	(p<0,05)	(p>0,05)	(p>0,05)	(p<0,05)	(p<0,05)
September 21, 2018	0,30	0,31	0,35	6,45	3,10	-2,64
May 18, 2018	(p<0,05)	(p<0,05)	(p<0,05)	(p<0,05)	(p<0,05)	(p<0,05)
September 21, 2018	0,36	0,35	0,43	-4,35	-3,02	-2,64
June 22, 2018	(p<0,05)	(p<0,05)	(p<0,05)	(p<0,05)	(p<0,05)	(p<0,05)
MIB						
July 20, 2018	0,22	0,31	0,29	1,35	-9,73	-6,34
June 22, 2018	(p<0,05)	(p<0,05)	(p<0,05)	(p>0,05)	(p<0,05)	(p<0,05)
September 21, 2018	0,34	0,27	0,20	-2,50	-6,54	-6,91
March 23, 2018	(p<0,05)	(p<0,05)	(p<0,05)	(p<0,05)	(p<0,05)	(p<0,05)
September 21, 2018	0,38	0,35	0,28	-6,56	-7,84	-3,65
May 18, 2018	(p<0,05)	(p<0,05)	(p<0,05)	(p<0,05)	(p<0,05)	(p<0,05)

Source: Authors' calculation using R Studio and Thomson Reuters data

From Table 1.2, we provide aggregate DM test results for all observed stock indices and combinations of maturity and trading dates. DM is used to test the null hypothesis for the observed pricing models having the same forecasting ability. We compare results obtained by the two-sided Kolmogorov-Smirnov test and the Diebold-Mariano test, respectively. As seen, in Table 1.2, the null hypothesis is rejected at a significance level of 5% in most cases except the DAX index on the trading date of March 23, 2018, and the maturity date of September 21, 2018. We also reject the null hypothesis of the Kolmogorov-Smirnov test in most of the cases that the estimated probability densities originate from the "true" density function. In the example of the AEX stock index on a maturity date of August 17, 2018, and trading date of June 22, 2018, the null hypothesis is at a significance level of 5% was rejected. In an equal number of cases, the Mixture Log-Normal model, the Shimko model, and the Edgeworth expansion model have been shown to have the same prognostic accuracy i.e. we did not reject the null hypothesis at a significance level of 5%. In the case of the DAX market index on the trading date of June 22, 2018, and the expiration date of July 20, 2018, all the models had the same prognostic accuracy. We did not reject the null hypothesis of the KS test at a significance level of 5% (p>0,05). This means that the probability density function implied by the Shimko model, and the "true" density function obtained by the Kernel estimator are the same. For each expiration date and every stock market index, the appropriate benchmark was found concerning Kernel bandwidth, and this benchmark was used for comparison purposes out-of-sample, and the estimated probability density which is sufficiently close to the "true" density, and thus it can be used as a benchmark or a reference function in determining the predictive accuracy of three option pricing models, taken in consideration.

We conclude that the probability density function can be effectively estimated from high-frequency data using the Kernel estimator. When the Kernel estimator is treated as a reference density, the Shimko model emerges as the best-fitting model out-of-sample when compared to the "true" density. Furthermore, the null hypothesis of the Kolmogorov-Smirnov test was rejected in most scenarios across all market indices and across various combinations of trading and expiration dates. The Diebold-Mariano test results did not reject the null hypothesis, indicating no significant difference in predictive accuracy among the models. Based on visual comparisons and the Kolmogorov-Smirnov test, we find that the Shimko model delivers the most precise forecasts. Therefore, it can be concluded that the Shimko model provides the best out-of-sample fit and offers the highest predictive accuracy among the evaluated models.

Our findings offer valuable insights for the development and refinement of predictive methodologies, particularly in the context of volatility estimation using high-frequency data—especially when specific characteristics, such as microstructure noise or irregular time intervals, are present. By understanding the behavior and structure of high-frequency financial data, researchers can enhance model accuracy and robustness. Furthermore, these results have practical implications for financial analysts who design forecasting techniques and define benchmark financial series, providing them with tools to improve their models' performance. In addition, the insights gained from this study can support investors and professionals in the Fintech sector by equipping them with more precise methods for tracking and interpreting trends in capital markets. This, in turn, facilitates more informed decision-making, better risk management, and the ability to respond proactively to rapidly changing market conditions.

Chapter 2

Is a robust two-times scaled estimator superior among competing alternatives of realized volatility?

Čuljak, M., Arnerić, J., & Žigman, A. (2022). Is Jump Robust Two Times Scaled Estimator Superior among Realized Volatility Competitors?. Mathematics, 10(12), 2124. pp. 11.

This research paper compares the developed estimators, among them the robust two-times scaled with the other four estimators. Fluctuations or volatility is an essential factor considered in finance volatility models. We also note that using high-frequency data based on the stored data of European stock markets for more than seven years is one of the crucial parts of our study. The following sources of data strengthen our data coverage with regard to market conditions and tendencies, which in turn increases the scope and richness of our investigation. It contributes not only to the theoretical and empirical literature related to volatility estimation with high-frequency data but also to the practitioner, who is seeking recommendations on which estimator should be applied in these markets depending on the degree of microstructure noise and price jumps. This benefits finance personnel because it enables them to come up with better decisions.

Within this framework, we establish that it is feasible to quantify the performance of these estimators and elucidate their substantial utility in informing decisions concerning the optimal frequency of data sampling. This is achieved through the application of Mincer-Zarnowitz regression analysis, Producer's Interval Tests (PIT-test), and the GoG upper tail dependence metric. The conclusions drawn herein are congruent with prior empirical findings and theoretical justifications, which underscore the importance of precise estimators in the formulation of market segmentation strategies and their role in enhancing volatility modeling. Volatility, by definition, encapsulates the rate at which the price of a financial asset or commodity fluctuates, rendering it particularly salient in financial analysis. In alignment with risk theory, a broad spectrum of models has been developed by scholars to characterize the price dynamics of specific assets or securities. The fourth research question interrogates whether alternative indicators or signals can be identified to capture price variation. Accordingly, the refined inquiry assesses whether the efficient two-times scaled estimator offers a robust comparative framework for integrated variance estimation relative to alternative methods, utilizing high-frequency data at one-second intervals over a longitudinal span of seven years.

We consider two groups of estimators: noise in the price process and provide estimators robust to microstructure noise and robust to price jumps. To assess the performance of the estimators, we implement them on the set of high-frequency data ranging from January 4, 2010, to April 28, 2017, extracted from the Germany, Italy, France, and the UK stock markets. The data's principal brief is that the stock markets are more liquid, most active during the day, and highly traded. The best sampling rate for each estimator is chosen based on the bias and variance of each estimator and adapted to the characteristics of the stock market index in question. The upper tail dependence originating from the Gumbel copula is assumed to be an adequate measure of pairwise comparison. The supremacy of a sound-to-times-scaled estimator concerning the optimal slow time-scale sampling frequency is proven for all the analyzed markets.

Modeling processes with strong persistence, extended memory, asymmetry, non-normality, and price jumps become more challenging due to many features that should be considered, for example, in the financial time series data. This makes estimation and forecasting of variance of price returns challenging. Literature suggests that Various approaches (parametric or nonparametric ones) can be used. According to Barndorff-Nielsen and Shephard (2002), the main issue is determining the prediction accuracy of realized variance when an actual variance of returns, the so-called integrated variance (IV), is unknown. According to Andersen et al. (2003), the RV converges in probability to IV (can be efficiently estimated by the realized variance (RV)). This implies that RV is a perfect measure of IV when high-frequency prices are observed continuously and without estimation errors. When prices are measured with microstructure noise, the realized variance may not obtain the same properties as consistency and asymptotic unbiasedness, Hansen and Lunde (2005). Microstructure noise captures various parts of the trading process: bid-ask bounces, discreteness of price changes, differences in trade sizes or informational content of price changes, and strategic components of the order flow. According to Bandi and Russell (2008), it has been shown that the highly sampled data was contaminated by microstructure noise. In addition, obtaining such data is still hard, so choosing an appropriate sampling frequency based on a trade-off between accuracy and potential biases (Ait-Sahalia et al. (2005), Hansen and Lunde (2005)). When the optimal sampling frequency is found, RV can be used as a measure of IV; the volatility estimators need to be robust to microstructure noise because of its presence in high-sampled datasets, Boudt and Zhang (2015). Therefore, a two times times-scaled estimator was introduced to overcome the microstructure noise when there are no price jumps (Zhang et al. (2005)).

In contemporary financial markets, there remains a lack of consensus regarding the most effective volatility estimator, representing a critical gap in the literature that this study aims to address. We investigate whether the robust two-times scaled estimator demonstrates superior performance relative to alternative volatility measures across four major developed European markets. The implications of our findings are potentially substantial for these markets and are anticipated with considerable interest. This research adopts the optimal sampling frequency at a lower temporal resolution, ensuring methodological alignment with the study's primary objective. We detail the methodological framework and dataset employed, present the empirical results derived from our analysis, and offer the conclusions drawn from these findings.

High-frequency data from Thomson Reuters Tick History covering January 4, 2010, to April 28, 2017, from four stock markets (Germany, Italy, France, and the UK) is considered. The root mean square error (RMSE) of the proposed estimator is used to define the optimal slow time scale sampling frequency, and the fast time scale sampling frequency of 1 second is determined in advance, according to data availability of the observed financial markets within the shortest, nonempty and equidistant intervals. The RMSE of each estimator was calculated as the sum of its squared bias and variance. Afterward, the RMSE was minimized concerning slow time scale frequency, corresponding to the number of subsamples.

Table (2.1), shows descriptive statistics of the data we use, the number of trading days, and several 1-second observations; intraday data taken into consideration was during trading hours from 9 a.m. until 5:30 p.m. for all the four stock markets (Germany, Italy, France, and the UK), the optimal sampling frequency, i.e., high-frequency data from January 4, 2010, to April 28, 2017. In Figure of Čuljak et al. (2022a), we consider the data frame used was from March 1, 2017, to March 31, 2017, to illustrate the eight realized volatility estimators for the German stock market until March 31, 2017.

Table 2.1 Description of high-frequency data from January 4, 2010, to April 28, 2017

European market	Stock index	No. of trading days	No. of 1 sec. observations	Optimal slow time scale
Italy	MIB	1862	42773317	10 sec.
Germany	DAX	1863	56650069	20 sec.
France	CAC	1878	4665980	13 sec.
UK	FTSE	1850	48920222	30 sec.

Source: Authors' based on Thomson Reuters data

This study examines a range of realized volatility estimators, including Realized Variance, Bipower Variation, the Minimized Block of Two Returns, the Medianized Block of Three Returns, the Average Sub-Sampled Realized Variance, the Two-Times Scaled Realized Variance, the Robust Two-Times Scaled Realized Variance, and the Hayashi-Yoshida Realized Variance, all considered as estimators of integrated variance. For comprehensive mathematical definitions of these estimators, refer to Čuljak et al. (2022a). Particular attention is given to the Hayashi-Yoshida Realized Variance (HYRV_t^Δ), a threshold-based estimator that incorporates a jump detection mechanism, denoted as I_i, to mitigate the influence of discontinuities. All estimators exhibit sensitivity to the choice of sampling frequency, necessitating careful consideration of optimal frequency criteria in the presence of market microstructure noise and price jumps. For an in-depth discussion of these criteria, see Bandi and Russell (2008) and Bandi & Aït-Sahalia et al. (2005). According to Boudt and Zhang (2015), the superiority of alternative realized volatility estimators is supported by simulation evidence demonstrating reduced bias and lower mean squared error.

We calculate the realized volatility estimates and $RTSRV_t^{\Delta,k,\theta}$ and then compared three methods (Mincer-Zarnowitz regression, probability integral transformation (PIT) test, and Gumbel copula upper tail dependence) to determine which volatility estimator best fits the benchmark. Mincer-

Zarnowitz regression is based on the overall performance of the realized volatility models. PIT test was performed as a density goodness of fit procedure to test how the examined realized volatility estimators perform in comparison to $RTSRV_t^{\Delta,k,\theta}$ upper tail dependence was used to examine what happens in the extreme values or tails. The $RTSRV_t^{\Delta,k,\theta}$ is defined as a benchmark because it is confirmed that $RTSRV_t^{\Delta,k,\theta}$ is robust to market microstructure noise jumps and non-synchronous trading in the intraday stock price series.

This study presents substantive contributions to the domain of financial market volatility estimation. We conduct a comparative analysis by evaluating seven realized volatility estimators against the benchmark of the robust two-times scaled estimator. For each of the selected stock markets (Germany, Italy, France, and the United Kingdom) the optimal sampling frequency is determined through the minimization of the root mean squared error (RMSE). These results are instrumental in enhancing the understanding of market volatility dynamics and informing data-driven decision-making processes. A fundamental consideration in this context is the inherent trade-off between estimator bias and statistical efficiency when selecting the appropriate sampling frequency. According to Ait-Sahalia et al. (2008) & Bandi and Russell (2008), one must establish the optimal sampling frequency for an observed financial market to reduce the bias for a volatility estimator to remain efficient. However, in the presence of jumps, there will also be bias in practical applications of estimators that are not jump-robust. So, it will influence sampling frequency, which is also influenced by market structure, liquidity, and microstructure noise. Ait-Sahalia et al. (2008) and Andersen et al. (2012) suggest that the way to increase the efficiency of estimators is to sub-sample (taking the average of an estimator across all possible sub-samples). Since we get the maximum amount of information from ultra-high frequency data, we use returns sampled as often as possible to calculate the RV. However, the literature suggests that if the sampling frequency is as high as it can be, it leads to a bias problem due to microstructure noise. We minimize the RMSE against the number of subsamples to obtain the optimal sampling frequency (optimal slow time scale). We find that the optimal sampling frequency for Germany is 20 seconds; for the UK, it is 30 seconds; for Italy, it is 10 seconds; and for France, it is 13 seconds.

We compare three methods (Mincer-Zarnowitz regression, probability integral transformation (PIT) test, and Gumbel copula upper tail dependence). The results of the first two methods for comparison of the realized volatility estimates indicate that. RV_t^{Δ}, $TSRV_t^{\Delta,k}$, $ARV_t^{\Delta,k}$ and $HYRV_t^{\Delta}$ underestimate the performance of estimates during severe stress and price jumps. Therefore, the results do not answer which estimator fits the benchmark best. In that case, upper tail dependence is a favorable method because it considers extreme values. The Mincer-Zarnowitz regression tested how well the volatility estimators fit. $RTSRV_t^{\Delta,k,\theta}$.

Table (2.2) gives the chi-squared test statistics and p-value results. For four stock markets (Germany, Italy, France, and the UK) and seven volatility estimators, the null hypothesis was rejected at a significance level of 5%. This showed how seven observed volatility estimators do not fit well with the h $RTSRV_t^{\Delta,k,\theta}$.

Table 2.2 Results of Mincer-Zarnowitz regression

	$MedRV_t^{\Delta}$	$MinRV_t^{\Delta}$	RV_t^{Δ}	BPV_t^{Δ}	$TSRV_t^{\Delta,k}$	$ARV_t^{\Delta,k}$	$HYRV_t^{\Delta}$
MIB							
test statistic	3449.3***	2298***	8375.1***	5442.6***	8841.3***	7530***	8274***
β_0	0.000002	0.000002	0.000008	0.000001	0.000007	0.000007	0.000008
β_1	0.779631	0.799547	0.508353	0.739918	0.504905	0.518399	0.508959
DAX							
test statistic	2281.7***	1853.3***	6517.3***	3404.7***	8001.1***	6732.8***	6517.1***
β_0	-0.000002	-0.000001	-0.000008	-0.000003	-0.000009	-0.000009	-0.000008
β_1	0.850240	0.856405	0.766997	0.820539	0.763453	0.782201	0.766998
CAC							
test statistic	2359.3***	3739***	4611.3***	2776.1***	1853.2***	679.25***	470.67***
β_0	0.000010	0.000011	0.000004	0.000008	0.000002	0.000005	0.000005
β_1	1.275828	1.426721	1.006286	1.355853	0.792073	1.012190	1.006807
FTSE							
test statistic	4296***	4039.8***	7845.9***	5365.1***	9819.5***	8446.9***	7845.7***
β_0	0.000001	0.000001	-0.000002	-0.000001	-0.000002	-0.000002	-0.000003
β_1	0.782853	0.777687	0.724623	0.769927	0.713995	0.730009	0.724622

Note: ***, **, * represent the significance of the chi-squared test at the 1%, 5% and 10% level.
Source: Authors' calculation using R Studio and Thomson Reuters data

Additionally, we conducted the Kolmogorov-Smirnov test to evaluate the uniformity of the distribution and corresponding p-values. The results, summarized in Table (2.3), pertain to four major stock markets (Germany, Italy, France, and the United Kingdom) and encompass all considered volatility estimators. At the 5% significance level, the null hypothesis of uniformity was consistently rejected. These outcomes are corroborated by the results obtained from the chi-squared test, both in terms of test statistics and p-values, thereby reinforcing the robustness and reliability of our empirical findings. The probability integral transformation (PIT) test was used to check whether the difference between $RTSRV_t^{\Delta,k,\theta}$ our results show that other competing volatility estimators are uniformly distributed.

Table 2.3 Results of the PIT test

	$MedRV_t^{\Delta}$	$MinRV_t^{\Delta}$	RV_t^{Δ}	BPV_t^{Δ}	$TSRV_t^{\Delta,k}$	$ARV_t^{\Delta,k}$	$HYRV_t^{\Delta}$
MIB test statistic	14399.1***	16639.3***	9577.5***	14373.9***	8801.2***	10161.2***	9554.1***
DAX test statistic	15123.4***	15272.9***	12953.1**	16349.2**	10790.7***	11598.8***	12955***
CAC test statistic	10636**	10152***	11580**	10692***	17964***	11624***	11579***
FTSE test statistic	20727**	12495***	13824**	18095***	14016***	13099***	13824***

Note: ***, **, * represent the significance of the Kolmogorov-Smirnov test at the 1%, 5% and 10% level.

Source: Authors' calculation using R Studio and Thomson Reuters data

In Table (2.4), we present our results by utilizing the upper tail dependence coefficient and the Gumbel copula function, the tail dependence coefficient λ based on the Gumbel copula function, and δ that regulates the degree of reliance. The results indicate that for Italy, Germany and the UK, $RTSRV_t^{\Delta,k,\theta}$ has the highest upper-tail dependence with $MedRV_t^{\Delta}$, $MinRV_t^{\Delta}$ and BPV_t^{Δ} volatility estimators.

These results agree with Andersen et al. (2012) and Barndorff-Nielsen and Shephard (2006) findings that state that among all the competing estimators, only jump robust ones have produced almost similar volatility estimates as $RTSRV_t^{\Delta,k,\theta}$. In the case of France, $RTSRV_t^{\Delta,k,\theta}$ is best fitted with $TSRV_t^{\Delta,k}$, BPV_t^{Δ}, $ARV_t^{\Delta,k}$ and $MedRV_t^{\Delta}$ volatility estimators. As $TSRV_t^{\Delta,k}$ is robust to microstructure noise, it is no surprise that estimates are as good as $RTSRV_t^{\Delta,k,\theta}$.

Table 2.4 Upper tail dependence results

	$MedRV_t^{\Delta}$	$MinRV_t^{\Delta}$	RV_t^{Δ}	BPV_t^{Δ}	$TSRV_t^{\Delta,k}$	$ARV_t^{\Delta,k}$	$HYRV_t^{\Delta}$
MIB							
δ	7.09	6.81	5.35	6.72	5.23	5.51	5.37
λ	0.859	0.853	0.813	0.851	0.809	0.819	0.814
DAX							
$CapDelta$	7.96	7.65	6.51	7.6	6.64	6.65	6.51
λ	0.874	0.869	0.846	0.868	0.849	0.85	0.846
CAC							
$CapDelta$	5.86	5.72	5.75	5.93	6.93	5.85	5.75
λ	0.829	0.825	0.826	0.831	0.856	0.829	0.826
FTSE							
$Delta\delta$	7.34	7.02	5.69	7.06	5.81	5.8	5.69
λ	0.864	0.858	0.824	0.858	0.828	0.828	0.824

Source: Authors' calculation using R Studio and Thomson Reuters data

We conclude that the benchmark is robust two times scaled realized variance ($RTSRV_t^{\Delta,k,\theta}$). As the realized variance (RV^{Δ}) is the most used high-frequency estimator, it is biased due to microstructure noise. The existing theory and scientific research on volatility estimators indicate that intraday data is needed for more precise volatility measures. We focused on observed intraday 1-second observation data to determine the optimal sampling frequency for the robust two-times scaled realized variance. It is from 10 to 30 seconds. The existing literature does not agree on the "best" realized volatility estimator. We determine whether the robust two-times scaled variance is the best. It is a superior volatility estimator for each of the four stock markets (Germany, Italy, France, and the UK) considered in this study. We compared the performance of two groups of estimators: those that are robust to microstructure noise and those that jump robust.

We used three methods (Mincer-Zarnowitz regression, probability integral transformation (PIT) test, and Gumbel copula upper tail dependence) to determine which volatility estimator fits best with the benchmark; the results indicated that the mediatized block of three returns ($MedRV^{\Delta}$) performed most similarly to the robust two times scaled realized variance ($RTSRV_t^{\Delta,k,\theta}$) The two times scaled for Italy, Germany, and the UK for France. Realized variance ($TSRV_t$) realized volatility estimator was the most similar (approximately equal) to the benchmark; we then conclude that the French financial market is more contaminated by microstructure noise than price jumps since the medianized block of three returns ($MedRV^{\Delta}$) is robust only to price jumps. Price jumps contaminate the Italian, German, and UK financial markets more than microstructure noise at selected sampling frequencies.

Moreover, the three evaluative techniques employed in this study - namely the Mincer-Zarnowitz regression, the Probability Integral Transformation (PIT) test, and the Gumbel copula upper tail dependence - were utilized to assess the alignment of various volatility estimators with the benchmark. Our findings indicate that the combined application of these methods provides a robust framework for benchmarking, offering valuable guidance to financial analysts and investors in selecting the most appropriate realized volatility estimator for specific market indices. This research addresses a notable gap in the literature concerning the computation of realized volatility estimators in developed European markets, even when high-frequency price data are available. Through our methodological approach, which emphasizes the selection of optimal low-frequency sampling tailored to each market and ensures robustness to price discontinuities, we successfully bridge this gap. This contribution significantly advances the existing body of research on volatility estimation.

A limitation encountered during the study was restricted access to comprehensive datasets, which posed challenges during the data collection phase. Future research could extend this work by examining realized covariances across asset classes, particularly in relation to correlation as a standardized measure of covariance, which is known to be sensitive to sampling frequency. Another promising direction involves exploring inter-asset relationships, such as those between bonds and equity indices, by analyzing the distinct characteristics of bonds and commodities and investigating their correlations with traditional financial instruments.

Chapter 3

Benefits of Sectoral Cryptocurrency Portfolio Optimization

Čuljak, M., Tomić, B., & Žiković, S. (2022). Benefits of sectoral cryptocurrency portfolio optimization. Research in international business and finance, 60, 101615. pp. 9.

A portfolio, defined as a collection of financial assets, represents one of the most effective instruments available to investors seeking to maximize returns. Accordingly, understanding the income dynamics of a selected portfolio enables investors to assess and estimate the associated investment risk. This study formally introduces and evaluates the advantages of sector-based cryptocurrency portfolio optimization, emphasizing its efficiency. To construct optimal portfolios, six distinct optimization models are employed: Minimum Variance (MinVar), Conditional Value at Risk Minimization (MinCVaR), Maximum Sharpe Ratio (MaxSR), Maximum STARR Ratio (MaxSTARR), Maximum Utility (MaxUT), and Maximum Mean Return (MaxMean). The average performance of these portfolios is benchmarked against the CRIX index, which reflects the overall performance of the cryptocurrency market during the same period. The results indicate that five of the six portfolio strategies demonstrate enhanced effectiveness when incorporating sector-specific cryptocurrencies, particularly from the financial, exchange, and business services sectors, thereby improving the investor's ability to achieve superior returns.

The rapid advancement of information technology and computational capabilities, coupled with growing mistrust in the global financial and payment infrastructure—exacerbated by recent economic crises and geopolitical tensions—has intensified interest in the development of digital currencies and alternative transaction mechanisms. In response, decentralized transaction systems that eliminate the need for intermediaries between transacting parties have emerged, as initially conceptualized by Nakamoto (2008). The launch of Bitcoin, the world's first cryptocurrency, on January 9, 2009, marked a pivotal moment in this evolution. It garnered significant attention from users due to its ability to facilitate near-instantaneous transactions with minimal fees, all without reliance on centralized authorities or intermediaries. For the creation of several new cryptocurrencies, and their characteristics, see, Čuljak et al. (2022). The availability of the cryptocurrency market despite its entire structure has been growing yearly. We've seen an increasing number of institutional and individual investors of different biographies investing and trading in cryptocurrencies, as numerous suppose cryptocurrencies are a licit asset class in investors' view. Investors apply different techniques, models, and strategies to construct their portfolio whose performance dynamics should outperform the market, that is, a portfolio that should yield more than market equilibrium returns. Such a definition entails the pursuit of undervalued assets, which would ultimately result in a market that is information-efficient, that is, a market whose aggregate value reflects all relevant and available information related to individual assets. When conventional definitions of investment are applied to the cryptocurrency market, notable inconsistencies emerge. Given the increasing prominence of cryptocurrencies in global finance, there is a pressing need for rigorous financial analysis and sustained academic inquiry. This study explores the interrelationships among cryptocurrencies and their respective sectors to construct and model optimized portfolios—specifically, those capable of outperforming the broader market. To evaluate this, we compare the performance of the constructed portfolios against the CRIX index, which serves as a benchmark representing the overall cryptocurrency market during the same time frame. The structure of the paper is as follows: we begin by outlining the data and methodological framework, followed by a presentation and interpretation of the empirical results obtained and at the end we will give concluding reflections on the findings of this paper.

To validate the algorithms, we use publicly available daily price data (in USD) for a total of 65 cryptocurrencies collected from the Coinmarketcap - CMC platform pages. We consider data for the 8/26/2019 to 02/22/2020 period, creating a sample of a total of 146 daily observations, or 145 daily returns for a 65-time series. This sample space is enough to create optimal portfolios for our data. For optimal portfolio composition and testing the utility of the cryptocurrency sectoral division, we consider an existing portfolio consisting of the top 50 cryptocurrencies by market capitalization including an additional 15 cryptocurrencies, and 5 leading cryptocurrencies by each of the three leading utilization sectors by market capitalization. We form multiple portfolios with different optimization goals of risk minimization, return maximization, and maximization of return and risk ratios. We use the conditional Value at Risk - CVaR as the risk measure, i.e. the methodology that follows the work of Rockafellar et al. (2000). Our optimization goals are as follows: minimum variance (MinVar), minimum CVaR (MinCVaR), maximize Sharpe ratio (MaxSR), maximize stable tail-adjusted return ratio (MaxSTARR), maximize utility function (MaxUT) and maximize mean return (MaxMean). We examined the advantages of treating the cryptocurrency market by sector segmentation. This involved including sector-specific cryptocurrencies in the portfolio and establishing linear group constraints. The constraints required 20% of the total portfolio allocation to sector-specific cryptocurrencies based on optimization goals. This was then compared with the optimal portfolios comprised of the top 50 cryptocurrencies by market capitalization. We then applied the standard portfolio optimization process to each respective asset allocation model. For the mathematical formulation of the asset allocation models, Performance Metrics i.e Sharpe ratio, MSquared, Regression alpha, Jensen's alpha, Treynor ratio, and Information ratio all used in this study, please refer to section 3.1 of Čuljak et al. (2022).

Table 3.1: Asset allocation models without sectors cryptocurrencies

		Asset Allocation Models						
Performance Metrics		MinVar	MinCVaR	MaxSR	MaxSTARR	MaxUT	MaxMean	CRIX
Beta	β_i	0,05	-0,05	0,02	-0,014	-0.05	-0,01	1
Annualized Alpha	a_{ai}	1,12	1,91	1,16	1,53	0,97	2,29	/
Annualized Return	R_{Gi}	0,94	1,44	0,95	0,62	0,72	0,95	0,57
Annualized Std Dev	σ_{ai}	0,49	0,54	0,48	0,94	0,46	1,04	0,47
Worst Drawdown	WD	0,27	0,29	0,26	0,57	0,27	0,56	0,31
Cumulative Return	CY	1,46	1,67	1,47	1,32	1,37	1,47	1,29
Sharpe Ratio	SR	1,92	2,68	1,95	0,66	1,58	0,91	1,20
MSquared	M^2	0,91	1,27	0,92	0,31	0,75	0,43	0,57
Treynor Ratio	TR	18,69	/	59,03	/	/	/	0,57
Jensen's Alpha	α_i	0,91	1,47	0,94	0,63	0,75	0,95	/
Information Ratio	IR	0,56	1,20	0,56	0,05	0,23	0,34	/

Source: Authors' calculation using R Studio

Based on the same performance measures, we first show the results of the portfolios formed by asset allocation models according to the initial selection of the portfolio components that is, for the six asset allocation models (without sectors cryptocurrencies) for 50 cryptocurrencies selected by CMC market size.

Table 3.2: Asset allocation models with sectors of cryptocurrencies

		Asset Allocation Models						
Performance Metrics		MinVar-S	MinCVaR-S	MaxSR-S	MaxSTARR-S	MaxUT-S	MaxMean-S	CRIX
Beta	β_i	0,05	0,03	-0,08	0,10	-0,12	0,22	1
Annualized Alpha	a_{ai}	0,86	2,39	1,37	3,74	1,51	10,10	/
Annualized Return	R_{Gi}	0,73	1,99	1,09	3,02	1,09	5,84	0,57
Annualized Std Dev	σ_{ai}	0,45	0,53	0,43	0,67	0,48	1,17	0,47
Worst Drawdown	WD	0,27	0,22	0,29	0,23	0,25	0,35	0,31
Cumulative Return	CY	1,37	1,88	1,52	2,23	1,53	3,02	1,29
Sharpe Ratio	SR	1,62	3,79	2,53	4,50	2,27	4,99	1,20
MSquared	M^2	0,77	1,79	1,20	2,13	1,07	2,36	0,57
Treynor Ratio	TR	14,91	76,54	/	31,22	/	26,88	0,57
Jensen's Alpha	α_i	0,70	1,98	1,12	2,97	1,16	5,72	/
Information Ratio	IR	0,26	2,04	0,77	3,09	0,74	4,31	/

Source: Authors’ calculation using R Studio

As shown in Table 3.1, our results show that the MinCVaR portfolio has the best values, with a high Sharpe Ratio (SR) of 2.68 which is by far the best among all other optimization strategies. This is not surprising since the primary goal of the MinCVaR portfolio is to minimize CVaR. It is important to note that the difference between the MSquared return and the risk ratio relative to the CRIX index also benefits the portfolio, which minimizes the conditional value at risk, as of Čuljak et al. (2022). On the other hand, by including an additional 15 sectoral cryptocurrencies that would not initially be selected as a component of the portfolios by their market capitalization, the results differ by all measures shown in Table 3.2. Based on the same performance measures and the same performance criterion, our results show that the return of the MaxMean-S portfolio adequately compensated for the higher risk assumed, which ultimately resulted in a high Sharpe Ratio (SR) of 4.99. MSquared also points out the difference between the MaxMean-S portfolio and the CRIX index and all the portfolios achieved a higher Sharpe ratio than the CRIX index during the same

period. As shown in Table 3.2, in the second order of the best size of all performances, except for the risk measures, it was achieved by a portfolio that maximizes the ratio of return and risk expressed as CVaR. As of Čuljak et al. (2022), the lowest worst drawdown belongs to the MinCVaR-S portfolio, which is in line with the optimization goal. In addition, Jensen's alpha suggests that all observed portfolios have yielded higher than expected returns per CAPM ratio. In comparison with the CRIX index, all the optimization goals achieved a higher cumulative return in the same observation period. As seen in Table 1 and Table 2 we observe the following. Portfolios with additional cryptocurrencies earn on average more returns than portfolios without sectoral components. The height of the regression alpha for all portfolios except the MinVar-S, achieved a better result if sectoral cryptocurrencies were included in the portfolio. Only the MinVar-S portfolio has a lower return than the portfolio returns without sectoral cryptocurrency, thus confirming our finding and logic that there are benefits in treating the cryptocurrency market through sectoral affiliation. In terms of risk, four strategies involving sectoral cryptocurrencies have achieved a lower standard deviation than portfolios without them. The inclusion of additional sector cryptocurrencies in existing portfolios contributes to the improvement of portfolio performance compared to the market represented by the CRIX index. Treynor ratio and Information ratio also performed significantly better for all sector portfolios except MinVar-S portfolios.
However, the higher risk was offset by the higher return achieved, implying a higher Sharpe ratio. A significant increase in cumulative return was also achieved by the MaxSTARR-S portfolio of 0.91, or 69%, compared to MaxSTARR. From our results in Table 1 and Table 2 and discussions, it is easy to see that the inclusion of additional sector cryptocurrencies in existing portfolios contributes to the improvement of portfolio performance compared to the market represented by the CRIX index. The geometric return has the same relationship. The inclusion of sector cryptocurrencies has also led to an increase in cumulative return for all strategies except the MinVar-S portfolio. The biggest difference was recorded by the MaxMean-S portfolio, where its cumulative return increased by 1.55. By applying a return maximization strategy and considering sectoral cryptocurrencies as a component of the portfolio, it was possible to achieve a cumulative return higher by 105% over 146 days than the same strategy that does not consider sectoral cryptocurrencies.

The analysis concludes that five out of the six portfolios constructed under distinct optimization objectives yielded superior performance when cryptocurrencies were categorized by sector—specifically within the financial, exchange, and business services domains. The findings suggest that selecting portfolio components based solely on market capitalization implies that such assets have already attained a valuation level that qualifies them for inclusion, thereby limiting their potential for further price appreciation. In contrast, cryptocurrencies with relatively low market capitalization present greater growth potential and are more readily identifiable when analyzed through a sectoral lens. These insights offer a meaningful contribution to the literature on investment strategies in the cryptocurrency market and the role of sectoral segmentation. Sector-

based analysis facilitates the early identification of emerging cryptocurrencies with lower capitalization. Moreover, evaluating the aggregate capitalization of specific sectors enables investors to detect prevailing market trends more effectively, as exemplified by the notable rise of DeFi-related cryptocurrencies in 2019.
We observe the existence of distinction within the category of cryptocurrencies and also the category of utilization tokens, that is if cryptocurrency bitcoin is used solely as a means of payment, comparing litecoin with a decentralized computer platform like Ethereum and treating the two assets equally in the context of investment opportunities, is not practical or even impossible and this also confirmed by our results. Previous research papers have considered cryptocurrencies as a homogenous asset and relied solely on the general market optimization algorithm when selecting portfolio components. Such an approach implies a consensus on the magnitude of the equilibrium expected return of the selected cryptocurrencies. However, previously it has been shown that such a return does not even exist due to the absence of adequate valuation, that is, the intrinsic value of cryptocurrencies. Considering previous research, if one draws a parallel between thinking of the traditional capital market and the CAPM model, it can be concluded that all cryptocurrencies are properly valued, i.e. all cryptocurrencies are on the Security Market Line-SML, however, our results in this paper suggest the opposite. Our conclusions are supported by positive average regression and realized alpha portfolios, as well as portfolios with additional sectoral cryptocurrencies. In line with the obtained results, our findings emphasize the utility and necessity of observing the cryptocurrency market by sectoral affiliation to find potentially "undervalued" cryptocurrencies. If portfolio components are selected solely by market capitalization, it would mean that these cryptocurrencies have already achieved the value that makes them a potential portfolio component.

This paper examines the impact of sectoral affiliation when constructing a portfolio by observing cryptocurrency utility. A new approach for optimal formation was implemented, that is, the methodology for exploring the benefits of sectoral allocation and portfolio construction, where we limited the performance of the portfolio in composition to market capitalization. We then consider portfolios with the cryptocurrencies of the three leading sectors by market capitalization that is: finance, exchanges, and business services. The results suggest that portfolios in which 20% of the weight is allocated to cryptocurrencies of lower market capitalization achieve higher values across all implemented performance measures in five of the six optimization strategies. It can be concluded that it is desirable and necessary to observe the cryptocurrency market through its type or its utility, and such an approach can be achieved by categorizing cryptocurrencies into their sectors. Prospective investors and portfolio managers in particular, should refrain from evaluating cryptocurrencies solely on the basis of market capitalization. This is due to the fact that cryptocurrencies possess distinct attributes and functionalities that are defined by their intended use. Portfolio managers are therefore encouraged to incorporate these assets into investment strategies by considering their intrinsic characteristics (such as type and functional purpose) when constructing portfolios. This approach not only mitigates the marginalization of certain cryptocurrencies but also enhances the overall performance potential of portfolios within the cryptocurrency market.

Chapter 4

Re-examining Safe Havens and Hedges through a Realized Covariance Lens

Čuljak, M., Arnerić, J., Žiković, S. & Salah Uddin, G. (2025) Re-examining Safe Havens and Hedges through a Realized Covariance Lens, paper in submission

During periods of economic and geopolitical instability, investment firms actively seek financial instruments that can preserve capital and yield returns despite heightened uncertainty. Safe-haven currencies are among such instruments, known for their resilience and performance under adverse conditions. Over recent decades, these assets have attracted considerable scholarly and practical interest, particularly in times of market turbulence. Investors typically gravitate toward safe-haven assets to secure stability and mitigate risk. This study investigates which potential safe-haven currency demonstrates superior performance over the observation period, using the Robust Two-Times Scaled Estimator of covolatility as the benchmark. Traditionally, currencies such as the U.S. Dollar, Swiss Franc, and Japanese Yen have been regarded as reliable stores of value during crises. However, the emergence of digital assets like Bitcoin has introduced new dynamics into the safe-haven asset landscape. This research seeks to determine which of these currencies, among the U.S. Dollar, Swiss Franc, Japanese Yen, and Bitcoin, exhibits the most favorable safe-haven characteristics over the period from June 2013 to May 2022, based on high-frequency (1-minute interval) data. By evaluating the performance of various realized covariance estimators on this dataset, we identify the most effective estimator and, subsequently, the best-performing safe-haven currency relative to broader market movements.

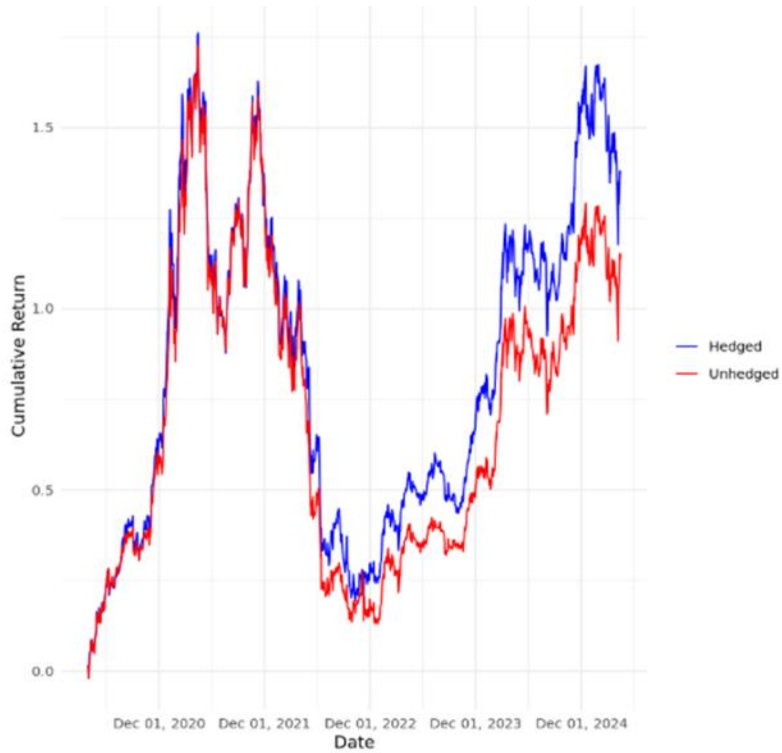
This study investigates which potential safe-haven currency demonstrates superior performance over the observation period, using the Robust Two-Times Scaled Estimator of covolatility in conjunction with various realized covariance estimators. Existing literature highlights a lack of consensus regarding the optimal realized covariance estimator, a gap this research aims to address. We identify the most effective covariance estimator for both synchronized and unsynchronized high-frequency financial data. This is accomplished by evaluating the performance of six covariance estimators across both data types, using established measures of model fit. Furthermore, we assess the influence of price jumps on each estimator's performance. The insights gained from this analysis are then applied to real-world high-frequency data to determine which asset class, among traditional currencies and emerging digital assets, exhibits stronger safe-haven characteristics.

This research utilizes realized covariance estimators. The two populations' integrated covariance (true but unknown) is estimated using high-frequency data. Several approaches do not use high-frequency data, such as historical covolatility (estimating the covariance by one number based on the selected observation period) and heteroscedastic covolatility (estimating the time-varying covariance using multivariate GARCH models). However, in this research, estimators of realized covariance are used for the same reason. The major challenge in choosing the most appropriate estimator of realized covariance is the non-synchronization of prices observed at unequal intervals. To address this, we employ a robust covariance estimation method that can handle such non-synchronization issues. There is no strict consensus on the superiority of the realized covariance estimator, so the results of this research paper will make a significant contribution because they will suggest which estimators are appropriate for each analyzed market and how to solve the problem of microstructural noise, price jumps, and non-synchronization, which are closely related to the selection of the optimal sampling frequencies on both the slow and fast time scale.

As there is no strict consensus on the superiority of the realized covariance estimator,
according to Ozer-Imer and Ozkan (2014), there is an inverse relationship between volatility and the duration of the crisis. Cho and Heejonn (2021) state that currencies such as the Swiss Franc and the Japanese Yen also exhibit safe haven characteristics. Fatum and Yamamoto (2016), in their study on the behavior of safe haven currencies during the global financial crisis from 2008 to 2009, found that the Swiss Franc and the Japanese Yen exhibited safe haven characteristics and acted as a source of stability and were sought after by investors seeking refuge in time of financial distress. Even though recent literature has concluded that Bitcoin and the Japanese Yen specifically have characteristics of a safe haven compared to traditional assets such as stocks and bonds; there is no resolution on which asset is "more" safe haven. Therefore, we show which asset is a better safe haven than traditional assets, such as stocks. We seek to understand the behavior of digital currencies during turbulent market conditions and for policymakers interested in the implications of cryptocurrencies for financial stability by using robust covariance estimation on the observed assets.

This study also investigates the role of the Swiss Franc (CHF) as a dynamic hedge for a diversified global investment portfolio. The portfolio consists of U.S. equities (S&P 500), European equities (DAX), emerging markets (EM), Bitcoin (BTC), and gold. Historical daily price data from January 1, 2020, to the present was used to calculate daily logarithmic returns for each asset. The portfolio was constructed using fixed weights: 25% S&P 500, 25% DAX, 20% EM, 20% Bitcoin, and 10% gold. CHF returns were analyzed separately as a potential hedging instrument. To model a dynamic hedge, a rolling 60-day window was applied to estimate the time-varying hedge ratio between the portfolio returns and CHF returns. The hedge ratio at each point in time was computed as the negative covariance between the portfolio and CHF returns divided by the variance of CHF returns over the same window. This dynamic hedge ratio was updated daily and applied to the portfolio to create a hedged return series. The performance of the hedged and unhedged portfolios was evaluated by comparing cumulative returns, annualized volatility, and annualized Sharpe ratios. Furthermore, the evolution of risk and risk-adjusted performance over time was visualized through 60-day rolling volatility and rolling Sharpe ratio plots. Cumulative returns for both the hedged and unhedged portfolios were plotted together on a single graph to facilitate a direct visual comparison as seen in Figure 4.1.

Figure 4.1 Comparison of Cumulative Returns between Hedged and Unhedged Portfolios



Source: Authors' calculation using R Studio

To determine the most effective covariance estimator for high-frequency datasets (including both synchronized and unsynchronized data) and ensure that the results are not biased and are reliable, we employ a simulation design to provide a comprehensive assessment of the accuracy of each estimator. This approach was proposed by Boudt and Zhang (2015) and Barndorff-Nielsen et al. (2011). To ensure our results are robust and reliable, we compare the performance of each estimator in different scenarios and identify any potential biases or limitations in their accuracy. Due to the usage of real-world 1-minute high-frequency data in the second phase of this research, for the simulation study, we set the fast time scale at one and the slow time scale at 20 for the realized covariance estimators. As we simulated 1-second data, the quick time scale was set as 1, and the slow time scale was defined by a similar design in Čuljak et al. (2022a), where optimal sampling frequency on a slow time scale was evaluated and recommended. In the simulation study, the estimators from Čuljak et al. (2022a) realized covariance version are compared to determine if the proposed has the best accuracy among alternative competitors of realized covariance.

Throughout the analysis, Synchronized means that the data was used immediately after the simulation without tempering the frequency between two-time series and using the default frequency of 1 second between the observations. The relative bias for most variance estimators increases as we increase or add jumps; this can be seen as overfitting and underfitting concepts in machine learning, as the increase in model parameters may lead to overfitting but also increasing bias. From Table 4.1, increases as the lowest relative bias had the Robust two times scaled estimator in the case when there were no jumps present in the simulated data; however, in the presence of jumps, the values of relative bias increase, showing that he accuracy of estimators decreases.

Table 4.1 Simulation results of synchronized high-frequency data

	Relative bias	RMSE
No jumps		
$rBPCov_t^{\Delta}$	-0.5462356	2.961614
$rCov_t^{\Delta}$	-0.5462497	2.961672
$rHYCov_t^{\Delta,\theta}$	-0.5462244	2.961672
$rThresholdCov_t^{k,h}$	-0.5462497	2.961672
$rTSCov_t^{\Delta,k}$	-0.1913763	2.162879
$rRTSCov_t^{\Delta,k,\theta}$	-0.1816072	2.154633
Small jumps		
$rBPCov_t^{\Delta}$	-0.6795248	3.552866
$rCov_t^{\Delta}$	-0.6796252	3.552007
$rHYCov_t^{\Delta,\theta}$	-0.6795684	3.552058
$rThresholdCov_t^{k,h}$	-0.6796252	3.552007
$rTSCov_t^{\Delta,k}$	-0.1934863	3.228447
$rRTSCov_t^{\Delta,k,\theta}$	-0.1837072	3.224062
Large jumps		
$rBPCov_t^{\Delta}$	-0.7455796	4.099145
$rCov_t^{\Delta}$	-0.7454552	4.099058
$rHYCov_t^{\Delta,\theta}$	-0.7453924	4.098981
$rThresholdCov_t^{k,h}$	-0.7454552	4.099058
$rTSCov_t^{\Delta,k}$	-0.1952669	3.329447
$rRTSCov_t^{\Delta,k,\theta}$	-0.1888425	3.325062

Source: Authors' calculation using R Studio

We observe that all estimators underestimate the actual value of integrated covariance; that is, when no jumps are present in the simulated high-frequency data, the results show that bias is less present with estimators. The RMSE of variance estimators increases as we include or increase jumps. This increase in RMSE indicates that the variance estimators are less accurate when jumps are present. With unsynchronized high-frequency data, the Relative bias increases as we include or increase jumps. Table 4.2 indicates that jumps impact the Relative bias, as Relative bias estimates increase with an increase or inclusion of jumps for both simulations of synchronized and unsynchronized high-frequency data. This result is the same as other measures of fit, such as root mean squared error (RMSE), though the associated estimated for RMSE or unsynchronized high-frequency data with or without jumps do not differ very much.

However, this is not a surprise since, in general, the Relative bias and RMSE as statistical properties and fit measures are different quantitatively. No evidence including jumps improves the covariance estimators in terms of lowering RMSE, as seen in synchronized and unsynchronized high-frequency data; the covariance estimators' RMSE increased as jumps are considered. We would opt for other measures of fit, like multiple R-squared Moreover, adjusted R-squared may give more statistical evidence as we include jumps in the model. Statistically, the closer the Multiple R-squared and Adjusted R-squared of the model to 1, the better the model. Furthermore, we performed a Chi-square test on safe haven to investigate characteristics from estimated negative correlation for each observed currency; our results show no statistical evidence to conclude that the proportions differ; this shows us that all of the examined currencies have characteristics of safe haven currency, as stated in the recent literature. Our simulation results of unsynchronized high-frequency data show that it is the best covariance estimator since it has a minor Relative bias for all cases. (with or without jumps). Simulation results of synchronized high-frequency data have the most minor Relative bias compared to other covariance estimators with or without jumps. This result supports our hypothesis since the associated RMSE is smaller than the RMSE of other covariance estimators; as jumps are included, these results show that it is valid to be defined as a benchmark and superior to other covariance estimators. We conducted a one-tailed Z-test to verify our results; we rejected the null hypothesis in the case of the Swiss Franc, where we can conclude that the proportion is significantly more significant than the specified tested value. Therefore, the Swiss Franc exhibits the best safe haven characteristics among competing alternative currencies.

Jumps had a profound influence on the relative bias across all estimators, leading to a decrease in accuracy (Nolte & Voev, 2007). The simulation results of synchronized high-frequency data, when extended to unsynchronized high-frequency data, revealed consistent trends in the impact of jumps on the relative bias and root mean squared error of the estimators. These findings underscore the significant influence of jumps on the accuracy of the estimators. It is therefore crucial to account for jumps when analyzing high-frequency financial data to ensure the reliability of covariance estimators.

Table 4.2 Simulation results of unsynchronized high-frequency data

	Relative bias	RMSE
No jumps		
$rBPCov_t^{\Delta}$	-1.05273	3.321425
$rCov_t^{\Delta}$	-1.052616	3.321758
$rHYCov_t^{\Delta,\theta}$	-1.05262	3.321736
$rThresholdCov_t^{k,h}$	-1.052616	3.321758
$rTSCov_t^{\Delta,k}$	-0.2568758	2.163979
$rRTSCov_t^{\Delta,k,\theta}$	-0.2479691	2.157633
Small jumps		
$rBPCov_t^{\Delta}$	1.246548	4.112031
$rCov_t^{\Delta}$	1.246545	4.111846
$rHYCov_t^{\Delta,\theta}$	1.247246	4.111849
$rThresholdCov_t^{k,h}$	1.246545	4.111846
$rTSCov_t^{\Delta,k}$	1.258619	2.254377
$rRTSCov_t^{\Delta,k,\theta}$	1.295133	2.247213
Large jumps		
$rBPCov_t^{\Delta}$	1.598908	4.626791
$rCov_t^{\Delta}$	1.599028	4.626593
$rHYCov_t^{\Delta,\theta}$	1.599747	4.626648
$rThresholdCov_t^{k,h}$	1.599028	4.626593
$rTSCov_t^{\Delta,k}$	1.368619	3.427033
$rRTSCov_t^{\Delta,k,\theta}$	1.296134	3.422213

Source: Authors' calculation using R Studio

The study compared six covariance estimators. $rBPCov_t^{\Delta}$, $rCov_t^{\Delta}$, $rHYCov_t^{\Delta,\theta}$, $rThresholdCov_t^{k,h}$, $rTSCov_t^{\Delta,k}$, and $rRTSCov_t^{\Delta,k,\theta}$, Across synchronized and unsynchronized high-frequency data, the Robust two-times-scaled estimator consistently exhibited the most minor relative bias, indicating its superior performance in estimating covariance. This significant finding, in line with existing literature, reinforces the efficacy of the Robust two-times-scaled estimator in high-frequency data analysis, providing a new level of understanding in this field.

In conclusion, the key findings of this study are as follows: Utilizing a high-frequency simulation framework, we evaluated the performance of various covariance estimators based on relative bias and root mean squared error. The results demonstrate the clear superiority of the Robust Two-Times Scaled Covariance Estimator. Furthermore, our analysis reveals that cryptocurrencies, particularly Bitcoin, exhibit safe-haven characteristics comparable to, and in some cases exceeding, those of traditional safe-haven currencies. Specifically, both Bitcoin and the Swiss Franc outperformed the Japanese Yen and the U.S. Dollar in terms of safe-haven behavior, as evidenced by their stronger negative correlations with general market movements.

Among the currencies analyzed, the Swiss Franc emerged as the most effective safe-haven asset, as indicated by its correlation with the benchmark estimator. This finding highlights the importance of further research into the role of cryptocurrencies in hedging strategies, a domain that presents both complexity and promise.

The study employed the Robust Two-Times Scaled Estimator to assess the safe-haven properties of Bitcoin, the Japanese Yen, the U.S. Dollar, and the Swiss Franc. The analysis of negative correlations with market benchmarks revealed that Bitcoin and the Swiss Franc demonstrated more pronounced inverse relationships, suggesting stronger safe-haven potential. Statistical testing further confirmed the Swiss Franc's superior performance in this regard. These insights offer valuable implications for investors, policymakers, and researchers seeking to navigate the evolving landscape of safe-haven assets in global financial markets.

3. CONCLUSION

The increasing degree of global economic interconnectedness has significantly altered the landscape of international financial markets. One notable consequence of this integration is the diminishing scope for achieving effective diversification across investment portfolios. As economies become more tightly interwoven, asset classes that were once considered largely uncorrelated tend to exhibit stronger co-movements during periods of market stress. This convergence undermines traditional portfolio theory, which relies on the assumption of imperfect correlation among assets to minimize risk through diversification. Consequently, the feasibility of constructing robust hedging strategies is severely compromised, particularly during episodes of systemic financial distress. This erosion of diversification opportunities is exacerbated by the heightened volatility that characterizes contemporary financial markets. Such volatility, combined with the ever-looming threat of financial contagion, where localized economic or financial disruptions propagate swiftly across borders, has intensified investor anxiety and prompted a global search for assets that can preserve value during turbulent times. In this context, the concept of "safe haven" assets has gained renewed prominence in both academic literature and practical investment strategy.

Safe haven assets are particularly compelling as a subject of scholarly inquiry due to their distinctive attribute: the capacity to retain or even increase in value during periods of acute financial instability. Their role as protective instruments against market downturns renders them especially attractive in the current macroeconomic environment. Within the broader universe of safe haven instruments, our research narrows its focus to a specific subset, safe haven currencies and digital assets. Among digital assets, Bitcoin stands out as the most prominent and widely recognized, often posited as a potential modern safe haven due to its decentralized nature and limited supply. By investigating these instruments, our study seeks to contribute to the growing body of knowledge surrounding asset behavior under conditions of financial duress and to assess their practical efficacy as hedging tools in a globally interconnected market.

The primary objective of this dissertation is to contribute to a deeper and more comprehensive understanding of critical financial market phenomena, with particular emphasis on the empirical behavior and interrelationships of key market indicators. Specifically, this research investigates the distributional properties of financial asset returns, the dynamics of volatility, the interdependence of volatilities across assets (covolatility), and the characteristics and performance of safe haven assets. By exploring these dimensions, the study seeks to provide robust insights into the mechanisms that govern market behavior under both normal and stress conditions. Such understanding is crucial for investors, policymakers, and academics aiming to develop more effective risk management strategies, portfolio diversification techniques, and crisis response mechanisms in an increasingly interconnected and uncertain global financial environment.

The first paper is of great interest for investment industry so that analysts know which option pricing model to use when assessing the market expectations. Hence, examines the investment posibilites which is one of the main objectives of this dissertation. The empirical analysis conducted in this study reveals that, in the majority of cases, we reject the null hypothesis of the Kolmogorov-Smirnov test, which posits that the estimated probability density functions are derived from the “true” underlying distribution. This statistical outcome indicates notable discrepancies between model-based and non-parametric estimations of return distributions. However, supplementary graphical analyses demonstrate that Kernel density estimation provides a sufficiently accurate approximation of the "true" density function, making it a viable benchmark for out-of-sample comparative evaluations. Among the various models examined, the Shimko model most consistently yields density estimates that closely align with those generated by the Kernel estimator. Specifically, in the case of the DAX index, the probability density function implied by the Shimko model is found to be statistically indistinguishable from the "true" density as captured by the non-parametric Kernel approach, underscoring the model’s relative robustness and empirical validity in this context.

The second paper offers a comprehensive empirical evaluation of various realized volatility estimators using high-frequency data from major European stock indices, aiming to identify the most reliable proxy for integrated variance in markets characterized by dense trading activity. While prior literature has predominantly focused on U.S. or simulated data, this paper addresses the distinct characteristics of European developed markets. Through rigorous pairwise comparison techniques, including probability integral transformation, Mincer-Zarnowitz regression, and copula-based upper tail correlation, the research demonstrates the clear empirical superiority of the Jump Robust Two Times Scaled ($RTSRV_t^{\Delta,k,\theta}$) estimator. Specifically, the $RTSRV_t^{\Delta,k,\theta}$ estimator consistently outperforms its competitors across optimal low-frequency intervals (10 to 30 seconds), offering robustness to both market microstructure noise and price jumps. The findings have practical implications for researchers, asset managers, institutional investors, and regulators by providing a robust and empirically validated benchmark for volatility estimation, thereby supporting more informed and sustainable financial decision-making.

The findings in the third paper derive from the applied methodological framework substantiate the relevance of sectoral analysis within the cryptocurrency market, offering novel insights into potential investment strategies. Specifically, the study demonstrates that portfolios incorporating a 20% allocation to sector-specific cryptocurrencies (particularly those with lower market capitalization) consistently outperform across all evaluated performance metrics. This empirical evidence not only validates the consideration of sectoral affiliation as a meaningful dimension for portfolio construction but also highlights the potential of such assets to enhance diversification and risk-adjusted returns. Moreover, the classification of cryptocurrencies by sector facilitates a more nuanced understanding of their roles as safe haven or hedging instruments under different market conditions. Consequently, the sectoral approach presents a justified and promising avenue for investors seeking to optimize exposure in the evolving digital asset space.

The conclusions of the fourth paper underscore the empirical superiority of the Robust Two Times Scaled Covariance estimator in accurately capturing asset co-movements, as evidenced by its lower relative bias and root mean squared error. Leveraging this estimator, the analysis reveals that Bitcoin exhibits notable safe haven characteristics, especially when compared to traditionally established safe haven currencies. However, despite Bitcoin's favorable performance, the Swiss franc consistently emerges as the most effective safe haven asset within the sample, demonstrating the weakest correlation with market benchmarks during periods of financial stress. These findings provide strong evidence that, while certain cryptocurrencies may serve as alternative safe haven instruments, conventional assets like the Swiss franc remain dominant in this role. This research thus contributes to the literature by validating Robust Two Times Scaled Covariance estimator as a reliable covariance measure in high-frequency settings and by offering a novelty perspective on the comparative safe haven potential of digital versus traditional currencies.

This dissertation was based on high-frequency data, enabling the establishment of benchmark values for both the Robust Two Times Scaled estimator of volatility and the corresponding covolatility estimator. In addition to methodological validation, the study investigated the behavior of potential safe haven assets and the performance of cryptocurrency-based portfolios, particularly during periods of financial distress. Empirical findings indicate that five out of six portfolio strategies yielded superior performance when incorporating sector-specific cryptocurrencies, particularly those within the financial, foreign exchange, and business services sectors. These results underscore the relevance of sectoral diversification within the cryptocurrency market and offer valuable insights for retail investors seeking to enhance portfolio performance through digital asset exposure. The study further highlights the need for future research into the effectiveness of cryptocurrencies as hedging instruments relative to traditional fiat currencies, particularly in the context of portfolio risk management.

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8. APPENDIX

PREDICTIVE ACCURACY OF OPTION PRICING MODELS CONSIDERING HIGH-FREQUENCY DATA

Abstract
Recently, a great attention has been given to forecasting, not only future expectations and the variance of financial time-series, but also the entire probability density function of the underlying asset. For that purpose, call and put European option prices are employed in this research which includes two steps. In the first step, several probability density functions are estimated using different option pricing models, considering data of major market indices with different maturities. These implied probability density functions are risk neutral at expiration date. In the second step, implied pdf's are compared against "true" density obtained from the high-frequency data to examine which one gives the best fit out-of-sample, i.e. which implied probability density function fits the "true" density most accurately ate expiration date. The "true" density function is unknown, but it can be estimated using high-frequency data adjusted for risk preferences. Therefore, the main objective of this research is to find a data driven benchmark of the "true" density function for major market indices in consideration. This research contributes to the existing literature in two ways: i) finding the benchmark of the "true" density function using high-frequency data within Kernel estimator and ii) determining the predictive accuracy of the option pricing models, which is the purpose of this research. The comparison of benchmark density function against estimated risk neutral probability functions produces applicative results for market participants and public authorities, respectively. Moreover, research cognitions are offer better insights into high-frequency data issues.

Keywords: option pricing models, high-frequency data, Kernel estimation, benchmark density function, predictive accuracy
JEL Classification: G12, G17, C02, C51, C58

1. Introduction
The objective of this research is to employ high-frequency data in determining the forecasting power of option pricing models. High-frequency data are used here to provide a reference probability density function that would be a benchmark for comparison purpose. The motivation of high-frequency data usage has been driven by technological advances in trading systems, and recording of almost every transaction that has been realized. The dominance of electronic trading in regulated markets, as well as multilateral trading is growing due to the development of algorithms, the increase in the dynamics of placing orders and liquidity. All the above-mentioned

factors provide record-breaking market activity at high-frequency leading to more information with high quality. The academic and practical interest for intraday data, observed at very short intervals of time, are market structures and trading processes that are subject to constant change. The inclusion of electronic trading platforms has automated and accelerated the execution of transactions, as well as trading reporting, and has enabled investors to automate their strategies, and managing orders in real time. The main purpose of this research is forecasting the future expectation, variance and higher moments of financial assets. Besides of high-frequency data observed every minute, the put and call options data on the stock market indices CAC (*Cotation Assistée en Continu*), AEX (*Amsterdam Exchange index*), MIB (*Milano Indice di Borsa*), and DAX (*Deutscher Aktien index*) are considered in this study. All of these data were obtained from the Thomson Reuters financial service. The research is conducted in two phases. The first phase includes estimating the implied probability density functions at the expiration date using options data. The second phase deals with comparing the estimated probability density functions against the reference density function based on high-frequency data, obtained by Kernel estimation method. Employing the Kernel density estimation method on observed high-frequency data in real time, provides an applicative contribution and thus a great advantage over other studies which mostly rely on simulation data. The used models are from a class of non-parametric, parametric and semi-parametric option pricing models: Shimko model, Mixture Log-Normal model and Edgeworth expansion model, respectively. The main objective of the research is to evaluate their predictive accuracy and to select the most appropriate one, not only that best fits the data, but has the highest predictive accuracy simultaneously. There are some issues regarding the high-frequency data, such as financial market illiquidity, and thus lack of data and doubts about sampling frequency selection since time intervals are required to be equidistant and non-overlapping. Comparing the reference probability density function with an estimated risk neutral density function results in recommendations not only for academics, but also practitioners, particularly for financial analysts and market participants, as additional information of the risk preferences can be found.

2. Literature review

It is already documented in the previous studies that current prices of financial instruments reflect information about future expectations and other moments (Bouden, 2007). Option prices give an insight into the expected value of the underlying asset under the assumption of risk neutrality, which makes option prices suitable for estimating the implied probability density function (Čuljak, 2019). Syrdal (2002) presents the first application of risk neutral probability density function on Norwegian option market and Sun (2013) explores application on SP 500 index options, whereas Santos (2011), Bliss and Panigirtzoglou (2002) and Šestanović et al. (2018) study risk neutral probability density function on European options data. For market participants, the appeal of using the implied probability density function relies on the ability to estimate probabilities in a series of future events, using market perceptions over a period of time. Market analysts and decision makers use this source of information to analyze market sentiment, uncertainty and extreme events, which are commonly embedded in interest rates and exchange rates (Bauwens et al., 2008). It has been shown that a set of option prices, call and put respectively, with the same maturity, but with different strike prices, can be used to extract the entire probability distribution of the underlying asset at expiration date (Banz and Miller, 1978; Breeden and Litzenberger, 1978). Previous studies have used different approaches, i.e. different option pricing models (Bouden, 2007; Liu et al., 2007; Lai, 2014). Despite being the best known and commonly used, the Black and Scholes model assumes that a log-normal distribution is not always applicable in practice. To address this issue, different methods have been proposed to extract risk-neutral density (RND) functions and to examine their robustness and forecasting power (Santos and Guerra, 2015). Therefore, it is necessary to investigate more closely other models that can be applied in practice. According to Jondeau et al. (2007) alternative option pricing models can be separated in two categories: structural and non-structural. Structural models assume specific price and/or volatility dynamics, while non-structural models allow the estimation of RND without assumptions about the price or volatility of the underlying asset (Šestanović, Arnerić and Aljinović, 2018). Non-structural models are divided into three categories: parametric, semi-parametric and non-parametric models, which have been considered in this paper also. Parametric models assume the form of a risk-neutral probability density function without prior assumptions about the underlying asset price dynamics (Čuljak, 2019). From the mentioned studies it can be easily concluded which model is the best fit in-the-sample, but it cannot be generally concluded which model predicts most accurately. Due to comparison purpose out-of-sample, predictive accuracy determination requires that probability density function of underlying asset is known. As it is unknown as well as related moments of the process that generates price dynamics, any attempt of comparison against assumed data generating process via simulation, leaves many doubts and issues. On the other hand, as time passes, we can observe what has happened at expiration date by taking high-frequencies, i.e. intraday observations at given maturity. This is exactly what this paper considers.

3. Implied risk neutral density estimation

This section presents three option pricing models employed in this paper: Mixture Log-Normal model (MLN), Edgeworth expansion model (EE) and Shimko model (SM). Also, this section provides an overview of option pricing that reduce the limitations of Black and Scholes model. The Mixture Log-Normal model is a parametric one. According to Bahra (1997), using the combination of two log-normal distributions, the expressions for the European call and put option price are obtained:

$$C(X,\tau) = e^{-r\tau}w[e^{\alpha_1+\frac{1}{2}\beta_1^2}\Phi(d_1) - \mathrm{X}\Phi(d_2)] + (1-w)[e^{\alpha_1+\frac{1}{2}\beta_1^2}\Phi(d_3) - X\Phi(d_4)] \quad (1)$$

$P(X,\tau) = e^{-r\tau} w[-e^{\alpha_1+\frac{1}{2}\beta_1^2}\Phi(-d_1) - X\Phi(-d_2)] + (1-w)[-e^{\alpha_1+\frac{1}{2}\beta_1^2}\Phi(-d_3) - X\Phi(-d_4)]$,
(2)

where

$$d_1 = \frac{-\ln(X) + \alpha_1 + \beta_1^2}{\beta_1}$$

$$d_2 = d_1 - \beta_1$$

$$d_3 = \frac{-\ln(X) + \alpha_2 + \beta_2^2}{\beta_2}$$

$$d_4 = d_3 - \beta_2.$$

The exercise price is X, τ is time until expiration, r is a risk-free rate, w is the weight which should be estimated along with parameters α_i, β_i for $i = 1,2$, whie Φ is a standard normal cumulative distribution function. It is assumed that the underlying asset price distribution is approximately log-normal, hence a weighted sum of log-normal density functions is used (Santos and Guerra, 2015):

$q(S_t) = \sum_{i=1}^{k}[w_i L(\alpha_i, \beta_i, S_t)]$, (3)

where $L(\alpha_i, \beta_i, S_t)$ is the log-normal distribution with parameters α_i and β_i , i.e. the location and the scale of each log-normal distribution.
To estimate parameters of the implied risk neutral density function it is necessary to solve the following minimization problem with respect to the vector θ that obtains the unknown parameters:

$$\min_{\theta} \sum_{i=1}^{n}[C_i(X_i,\tau) - \tilde{C}_i]^2 + \sum_{i=1}^{n}[P_i(X_i,\tau) - \tilde{P}_i]^2 + [we^{\alpha_1+\frac{1}{2}\beta_1^2} + (1-w)e^{\alpha_1+\frac{1}{2}\beta_1^2} - e^{r\tau}S]$$
(4)

After estimation of the parameters $\alpha_1, \alpha_2, \beta_1, \beta_2$ and w, the same estimates are used in (3) to obtain the implied risk neutral density function on maturity date T (Santos and Guerra, 2015).
Semi-parametric models suggest an approximation of the true risk-neutral probability density function. The semi-parametric model was used here as the Edgeworth expansion model. According to Jarrow and Rudd (1998) and Jondeau et al. (2007) the call option price expression for Edgeworth expansion is:

$C(Q) \approx C(L) + e^{-r\tau}(\gamma_{Q,1} - \gamma_{L,1})\frac{K_{L,2}^{\frac{3}{2}}}{3!}\frac{dl(X)}{dS_T} + e^{-r\tau}(\gamma_{Q,2} - \gamma_{L,2})\frac{K_{L,2}^2}{4!}\frac{d^2 l(X)}{dS_T^2}$. (5)

Using (5) it is easy to calculate implied mean, variance, skewness $\gamma_{Q,1}$ and kurtosis $\gamma_{Q,2}$. Expression for implied risk neutral density function is obtained by differentiating twice (5) with respect to the X and evaluating over S_T as follows:

$$q(S_T) = l(S_T) - \left(\gamma_{Q,1} - \gamma_{L,1}\right)\frac{K_{L,2}^{\frac{3}{2}}}{6}\frac{d^2 l(X)}{dS_T{}^3} + \left(\gamma_{Q,2} - \gamma_{L,2}\right)\frac{K_{L,2}^2}{24}\frac{d^4 l(X)}{dS_T^4}, \ (6)$$

where partial derivations are iteratively calculated.
Non-parametric models do not assume the form of a risk-neutral probability density function. The last model, which is the Shimko model used in this study, was a non-parametric one. The idea is primarily to gather all the information on the volatility curve by the polynomial $\sigma(X)$, the exercise price X, and then to use the Breeden-Litzenberger expression to estimate the probability density function (Breeden and Litzenberger, 1978; Jondeau et al., 2007; Shimko, 1993). Following expression for the European call option price is used:

$$C(t, S, X, T) = S\Phi\left(d_1\left(\sigma(\mathrm{X})\right)\right) - e^{-r\tau} X \Phi(d_2(\sigma(\mathrm{X}))). \ (7)$$

Finally, the expression for the implied risk neutral density function is obtained:

$$q(X) = e^{r\tau}\frac{\partial^2 C(t,S,\sigma(X),T)}{\partial X^2}. \ (8)$$

4. Kernel density estimation using high-frequency data

High-frequency data have some unique properties, which makes them challenging for practitioners as well as academics. One of the main features is that high-frequency data are not observed continuously, but rather discretely within not equally distant time points or an interval of time. High-frequency data are mostly nonnegative, positively autocorrelated with significant intraday periodicity and long memory and microstructure noise (Degiannakis and Floros, 2015; Florescu et al., 2016; Hautsch, 2012). Long memory refers to the decay rate of the statistical dependence of two points with an increase in the time interval, i.e. the linear dependence between two shifted data points decreases very slowly.
Microstructure noise property is a phenomenon observed with high-frequency data, and refers to the observed deviation of the price from the base price. The presence of the microstructure noise makes estimates of some parameters biased. It can be the result of various factors such as bid-ask differences, information asymmetries, price changes discreetness and order latency.
High-frequency data are also characterized by non-normality, i.e. they show the property of fat-tails. Many option pricing models, such as Black and Scholes, assume normality. However, in practice, it has been shown that unpredictable human behavior leads to extreme events and thus

non-normality. Abovementioned features make determining an appropriate distribution complex. However, finding a true, but unknown daily distribution based on intraday prices becomes possible using the Kernel density estimation method. It should be emphasized that the “true” probability density function can be obtained for each maturity date and then compared with ex-ante density functions derived from several option pricing models. Thus, we compare implied risk-neutral probability density functions and the estimated probability density functions as benchmarks of the “true” densities. Prior to the analysis, the raw data is cleaned as follows. It is assumed that, at each exercise price, call and put options are available in pairs. Cleaning is done by taking a sample that satisfies more criteria than required by the bid price to be greater than zero. Therefore, there is usually a big difference between the available call and put options prices and the actually used. If there are less than ten exercise prices for which we have call and put options, then the probability density function will not be estimated by any of the models used in this paper. The observed stock indices are CAC, AEX, MIB and DAX, i.e. the French, Dutch, Italian and German market index, respectively. Financial instruments used are call and put options on the major indices of the listed financial markets on combinations of options trading dates and options expiration dates in 2018 (Table 1).

Table 1 Options trading dates and expiration dates with respect to four stock market indices

Year 2018	Options expiration dates		
Options trading dates	July 20	August 17	September 21
March 23			AEX, DAX, MIB
April 20			AEX, CAC
May 18	DAX		AEX, CAC, DAX, MIB
June 22	AEX, CAC, DAX, MIB	AEX, DAX	AEX, DAX

Source: Thomson Reuters

From Table 1 it can be noticed that options data are not available for all market indices at given trading dates. For example, on trading date May 18, 2018 options data are available for all four indices AEX, CAC, DAX and MIB with expiration on September 21, 2018 (maturity horizon approximately 4 months), but only for DAX index with expiration on July 20, 2018 (maturity horizon one month). Further, we estimate the implied probability density functions on the expiration dates of the Shimko model, Mixture Log-Normal model, and Edgeworth expansion. Additionally to options data, high-frequency data were obtained from the Thomas Reuters database for the same expiration dates, for which kernel density method is used to estimate the “true” probability density functions.

4.1 Kernel estimation of probability density function

Probability density function $f(x)$ of a random sample $X_1, \ldots, X_n$ is usually unknown and should be estimated ex-post. The most well-known assumption free method is the kernel estimation:

$$f_\beta(x) = (n\beta)^{-1} \sum_{j=1}^{n} K(\frac{x - X_j}{\beta}) \text{ , (9)}$$

where the kernel function K (x) is a symmetric and unimodal probability density function and β is the bandwidth. Bandwidth controls the smoothness of the probability density function and impacts considerably the graphical presentation of the estimate itself regarding the skewness and kurtosis. The most commonly used kernel functions are: uniform, Epanechnikov and Gaussian. In our research we use a Gaussian kernel function, i.e. K(x) = φ (x), where φ is the standard normal probability density function. The choice of the kernel function has no significant effect on the final density estimation, i.e. studies have shown that the choice of the kernel function does not affect the outcome, while it is more sensitive to the bandwidth (Rosenberg and Engle, 2002; Arnerić, 2020). The Kernel estimator is intuitively very similar to the histogram methodology. Specifically, the Kernel primarily estimates the probability density function at each data point, and then sums all these densities to produce a final estimate. Comparing the obtained curve estimate with the histogram (using the same data) would threaten the obvious difference in the histogram smoothness and the Kernel estimate as the Kernel estimator converges faster to the true probability density function.

4.2 Bandwidth selection

Choosing an appropriate kernel bandwidth β is crucial in estimating the probability density function f(x). Many studies recommend some rule of thumbs but also arbitrary selection of β (Chiu, 1996). This paper employs kernel bandwidth for each expiration date separately. Although non-parametric Kernel estimation is now a standard technique in exploratory data analysis, there is still a great deal of controversy how to evaluate validity of an estimate and which kernel bandwidth is optimal. The main argument is whether to use integrated squared error or mean integrated squared error to select the optimal bandwidth. However, in this research the kernel bandwidth parameter is adjusted to obtain the best fit with respect to the high-frequency data for each expiration date under consideration.

5. Research results

This section provides results of comparison between estimated probability density functions, obtained by the three option pricing models, and benchmarks of the “true” probability density functions, obtained by Kernel estimation using high-frequency data. A graphical and analytical comparison is presented at each maturity date. The study that is most similar to our research compares three parametric density functions obtained by a mixture of two log-normal (MLN), Black-Scholes-Merton (BSM) and generalized beta (GB2) according to Arnerić et al. (2015). Mean square error (MSE) and absolute relative error (ARE) were used for pairwise comparison purpose only, neglecting the “true” probability density function that can be observed ex-post. Diebold - Mariano test (DM) is used to test which model has a lower MSE (Diebold et al., 1998). Abovementioned parametric models are usually overfitted making a wrong impression how these models fit the data. Due to unique characteristics of the proposed models we consider them to be sensitive to different maturity dates. Because semi-parametric and non-parametric approaches do not explicitly form the risk-neutral probability density function and there is no assumption about the function itself, this paper focuses on Shimko model (SM), Mixture Log-Normal model (MLN) and Edgeworth expansion model (EE). In our paper we implement out-of-sample comparison methods and determine predictive accuracy. Two tests were used here, the Diebold-Mariano test and the Kolmogorov-Smirnov test (Pauše, 1993). The results are provided for all combinations of selected trading and expiration dates. For the prices of call and put options the midpoints between bid and ask prices are taken. EURIBOR is taken as a risk-free interest rate, depending on the forecast horizon. The forecast horizon varies from 1 month to 6 months (Table 1). It was assumed that there were no dividend payments. Data processing was done in the “R Studio”. The results for the AEX, CAC, DAX and MIB index are presented graphically from Figure 1 to Figure 18 (see Appendix). The figures present graphical comparison of implied probability density functions using three different option pricing models (MLN, EE, SM) and Kernel estimated probability density function based on high-frequency data, i.e. the “true” density (TD). From these figures it can be observed that Kernel density estimation is accurate enough to be used as a benchmark for comparative purpose out-of-sample and that mostly Shimko model fits the “true” density the best. This result is of great interest for investment industry so that analysts know which option pricing model to use when assessing the market expectations. Table 2 presents the comparison results obtained by two-sided Kolomogorov- Smirnov test and the Diebold- Mariano test, respectively.

Table 2 Comparison results from the Kolmogorov-Smirnov test and Diebold-Maraino test

Index / Expiration date / Trading date	Kolmogorov-Smirnov test			Diebold-Maraino test		
	TD-MLN	TD-SM	TD-EE	TD-MLN	TD-SM	TD-EE
AEX						
August 17, 2018 June 22, 2018	0,26 (p<0,05)	0,54 (p<0,05)	0,67 (p<0,05)	-8,85 (p<0,05)	-7,51 (p<0,05)	-6,85 (p<0,05)
July 20, 2018 June 22, 2018	0,41 (p<0,05)	0,52 (p<0,05)	0,59 (p<0,05)	2,23 (p<0,05)	0,85 (p>0,05)	-2,21 (p<0,05)
September 21, 2018 March 23, 2018	0,38 (p<0,05)	0,61 (p<0,05)	0,72 (p<0,05)	-6,75 (p<0,05)	-7,81 (p<0,05)	-1,32 (p<0,05)
September 21, 2018 April 20, 2108	0,42 (p<0,05)	0,65 (p<0,05)	0,75 (p<0,05)	-5,74 (p<0,05)	-7,34 (p<0,05)	-4,24 (p<0,05)
September 21, 2018 May 18, 2018	0,37 (p<0,05)	0,59 (p<0,05)	0,65 (p<0,05)	-7,24 (p<0,05)	-8,21 (p<0,05)	-6,99 (p<0,05)
September 21, 2018 June 22, 2018	0,36 (p<0,05)	0,51 (p<0,05)	0,6 (p<0,05)	-5,41 (p<0,05)	-7,96 (p<0,05)	-7,38 (p<0,05)
CAC						
July 20, 2018 June 22, 2018	0,53 (p<0,05)	0,80 (p<0,05)	0,38 (p<0,05)	-4,06 (p<0,05)	-7,55 (p<0,05)	-0,87 (p<0,05)
September 21, 2018 April 20, 2018	0,54 (p<0,05)	0,47 (p<0,05)	0,54 (p<0,05)	-7,04 (p<0,05)	-8,62 (p<0,05)	-5,41 (p<0,05)
September 21, 2018 May 18, 2018	0,31 (p<0,05)	0,35 (p<0,05)	0,53 (p<0,05)	-7,99 (p<0,05)	-9,82 (p<0,05)	-8,58 (p<0,05)
August 17, 2018 June 22, 2018	0,35 (p<0,05)	0,47 (p<0,05)	0,53 (p<0,05)	-18,06 (p<0,05)	-10,94 (p<0,05)	-7,00 (p<0,05)
DAX						
July 20, 2018 May 18, 2018	0,37 (p<0,05)	0,55 (p<0,05)	0,38 (p<0,05)	-5,85 (p<0,05)	-1,45 (p>0,05)	-5,68 (p<0,05)
July 20, 2018 June 22, 2018	0,31 (p<0,05)	0,46 (p<0,05)	0,49 (p<0,05)	1,94 (p>0,05)	1,34 (p>0,05)	-0,76 (p<0,05)
September 21, 2018 March 23, 2018	0,34 (p<0,05)	0,15 (p<0,05)	0,13 (p>0,05)	0,34 (p>0,05)	-5,18 (p<0,05)	-7,70 (p<0,05)
September 21, 2018	0,30 (p<0,05)	0,31 (p<0,05)	0,35 (p<0,05)	6,45 (p<0,05)	3,10 (p<0,05)	-2,64 (p<0,05)

May 18, 2018						
September 21, 2018 June 22, 2018	0,36 (p<0,05)	0,35 (p<0,05)	0,43 (p<0,05)	-4,35 (p<0,05)	-3,02 (p<0,05)	-2,64 (p<0,05)
MIB						
July 20, 2018 June 22, 2018	0,22 (p<0,05)	0,31 (p<0,05)	0,29 (p<0,05)	1,35 (p>0,05)	-9,73 (p<0,05)	-6,34 (p<0,05)
September 21, 2018 March 23, 2018	0,34 (p<0,05)	0,27 (p<0,05)	0,20 (p<0,05)	-2,50 (p<0,05)	-6,54 (p<0,05)	-6,91 (p<0,05)
September 21, 2018 May 18, 2018	0,38 (p<0,05)	0,35 (p<0,05)	0,28 (p<0,05)	-6,56 (p<0,05)	-7,84 (p<0,05)	-3,65 (p<0,05)

Source: Authors calculation using R Studio and Thomson Reuters data.

In most of the cases we reject the null hypothesis of Kolmogorov-Smirnov test that the estimated probability densities originate from the “true” density function. The null hypothesis is rejected at a significance level of 5% in the most cases except from DAX index on trading date of March 23, 2018 and maturity date of September 21, 2018. In that case we did not reject the null hypothesis of KS test at a significance level of 5% (p>0,05). This means that probability density function implied by the Shimko model and the “true” density function obtained by the Kernel estimator are the same.

Table 2 also provides aggregate DM test results for all observed stock indices and combinations of maturity and trading dates. DM is used to test the null hypothesis for the observed pricing models having the same forecasting ability. In the example of AEX stock index on maturity date August 17, 2018 and trading date June 22, 2018 the null hypothesis at a significance level of 5% was rejected. In an equal number of cases, the Mixture Log-Normal model, the Shimko model, and Edgeworth expansion model have been shown to have the same prognostic accuracy i.e. we did not reject the null hypothesis at a significance level of 5%. It is important to emphasize that in the case of DAX market index on the trading date of June 22, 2018 and expiration date July 20, 2018 all the models had the same prognostic accuracy.

6. Conclusion

Probability density function can be estimated using high-frequency data by employing Kernel estimator. Estimated probability density is sufficiently close to the “true” density and thus it can be used as a benchmark or a reference function in determining the predictive accuracy of three option pricing models, taken in consideration. Finding a benchmark for comparison purpose out-of-sample is the main contribution of this research, i.e. for each expiration date and every stock market index, appropriate benchmark was found with respect to Kernel bandwith.

From the perspective that Kernel estimator provides referential probability density function, it can be concluded that Shimko model is the best fitting model out-of-sample when compared against the "true" density. Moreover, the null hypothesis of Kolmogorvo-Smirnov test was rejected in the most cases for all market indices and all combinations of trading and expiration dates. The results of the Diebold - Mariano test did not reject the null hypothesis implying that the models have the same predictive accuracy. According to the graphical presentations and the Kolomogorov-Smirnov test, we can conclude that the Shimko model predicts most accurately. Comparing a benchmark density function with an estimated risk-neutral density function has shown solid results that provide recommendations for application in academic research as well as in the financial industry. These results can be very helpful for further research in volatility estimation using high-frequency data. Concerning the application of the expected results it will be possible to observe certain characteristics within the development of predictive methods and their optimization. Contribution is made to analysts and investors of "Fintech" areas to whom have been proposed forecasting methodology and the reference financial series for monitoring future developments on the capital markets.

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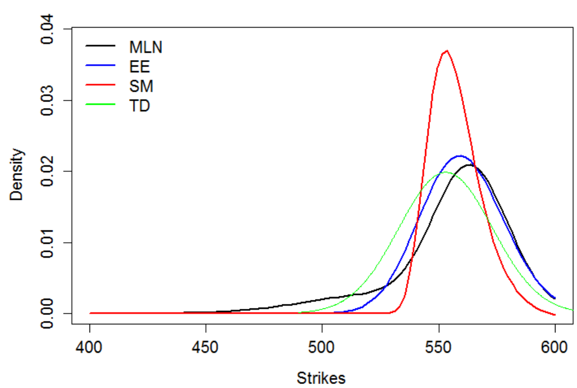
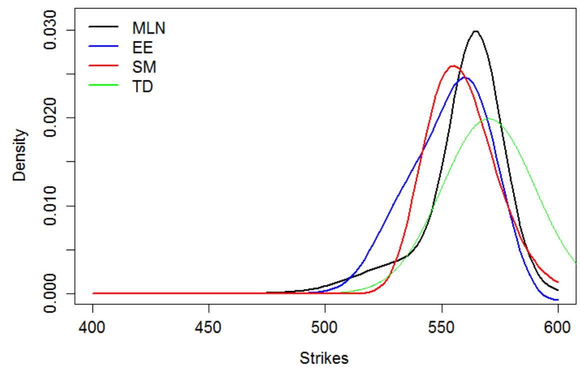
Appendix

Figure 1 Comparison of risk-neutral densities obtained on June 22, 2018 for AEX index towards the true density of AEX index with a maturity date of August 17, 2018



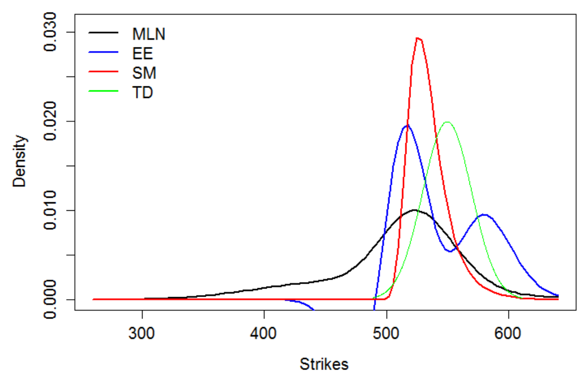
Source: Authors calculation using R Studio and Thomson Reuters data.

Figure 2 Comparison of risk-neutral densities obtained at June 22, 2018 for AEX index towards the true density of AEX index with a maturity date of July 20, 2018



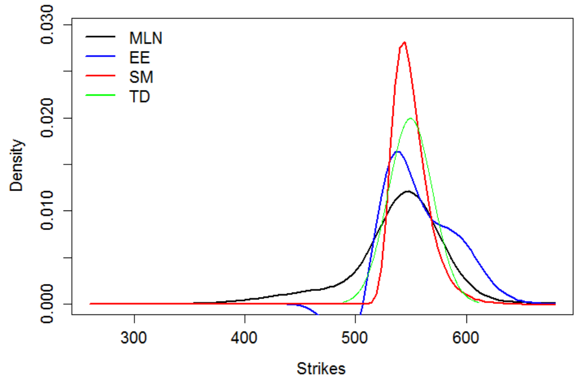
Source: Authors calculation using R Studio and Thomson Reuters data.

Figure 3 Comparison of risk-neutral densities obtained on March 23, 2018 for AEX index towards the true density of AEX index with a maturity date of September 21, 2018



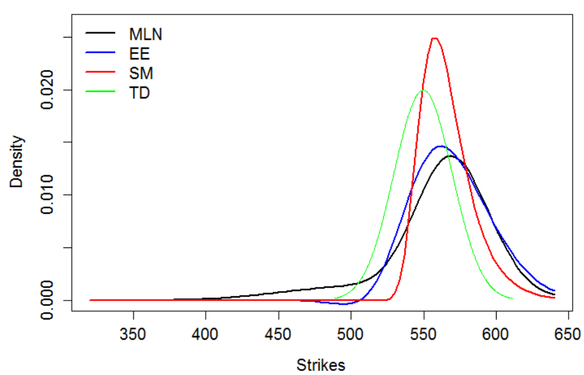
Source: Authors calculation using R Studio and Thomson Reuters data.

Figure 4 Comparison of risk-neutral densities obtained on April 20, 2018 for AEX index towards the true density of AEX index with a maturity date of September 21, 2018



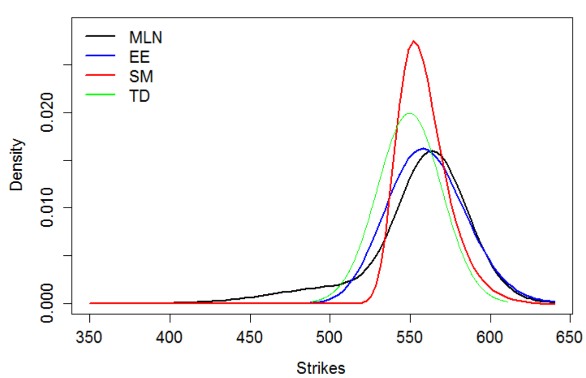
Source: Authors calculation using R Studio and Thomson Reuters data.

Figure 5 Comparison of risk-neutral densities obtained on May 18, 2018 for AEX index towards the true density of AEX index with a maturity date of September 21, 2018



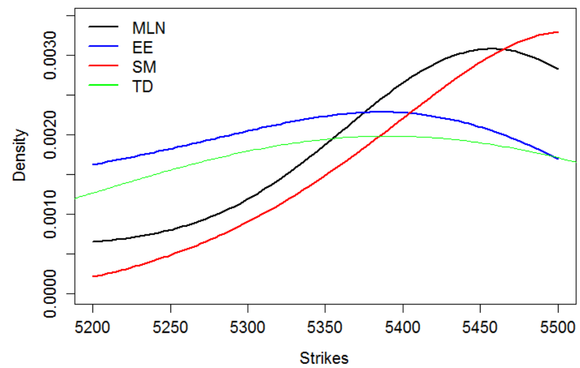
Source: Authors calculation using R Studio and Thomson Reuters data.

Figure 6 Comparison of risk-neutral densities obtained on June 22, 2018 for AEX index towards the true density of AEX index with a maturity date of September 21, 2018



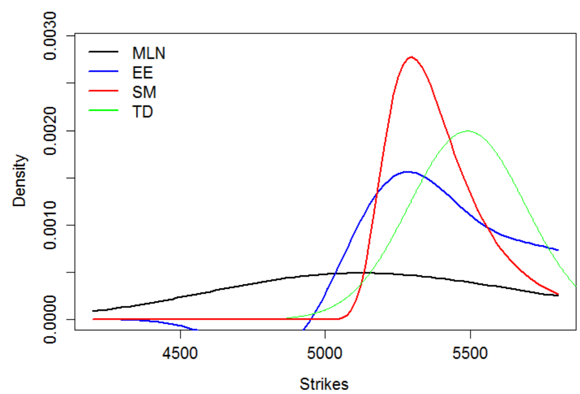
Source: Authors calculation using R Studio and Thomson Reuters data.

Figure 7 Comparison of risk-neutral densities obtained on June 22, 2018 for CAC index towards the true density of CAC index with a maturity date of July 20, 2018



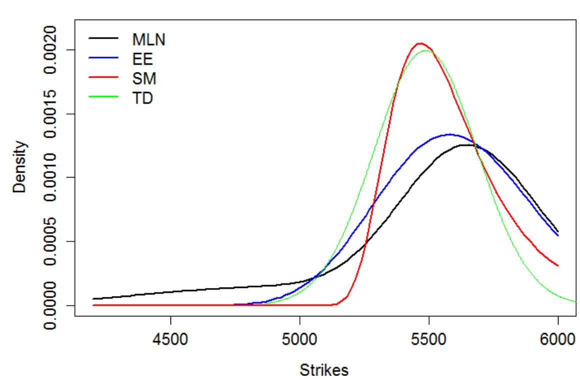
Source: Authors calculation using R Studio and Thomson Reuters data.

Figure 8 Comparison of risk-neutral densities obtained on April 20, 2018 for CAC index towards the true density of CAC index with a maturity date of September 21, 2018



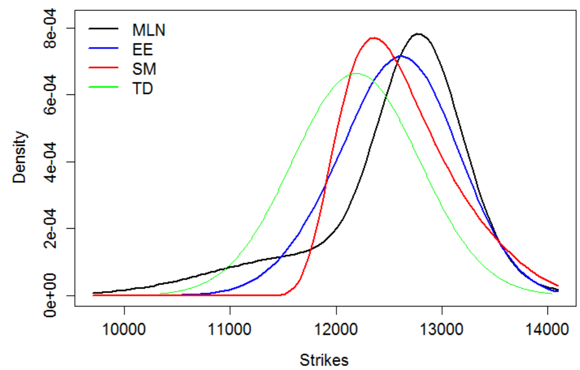
Source: Authors calculation using R Studio and Thomson Reuters data.

Figure 9 Comparison of risk-neutral densities obtained on May 18, 2018 for CAC index towards the true density of CAC index with a maturity date of September 21, 2018



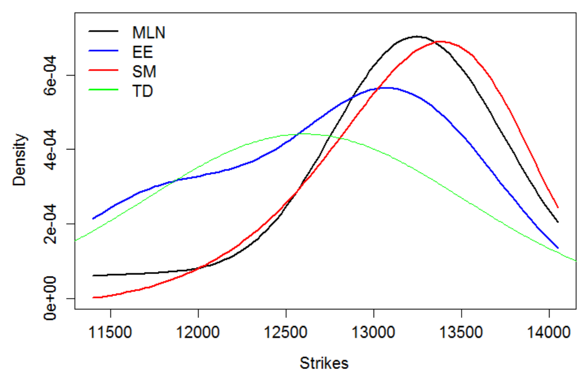
Source: Authors calculation using R Studio and Thomson Reuters data.

Figure 10 Comparison of risk-neutral densities obtained on June 22, 2018 for DAX index towards the true density of DAX index with a maturity date of August 17, 2018



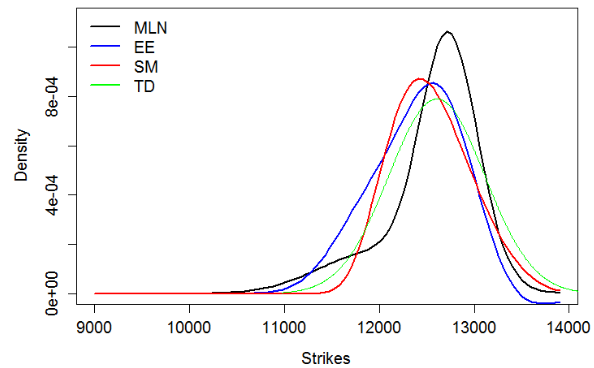
Source: Authors calculation using R Studio and Thomson Reuters data.

Figure 11 Comparison of risk-neutral densities obtained on May 18, 2018 for DAX index towards the true density of DAX index with a maturity date of July 20, 2018



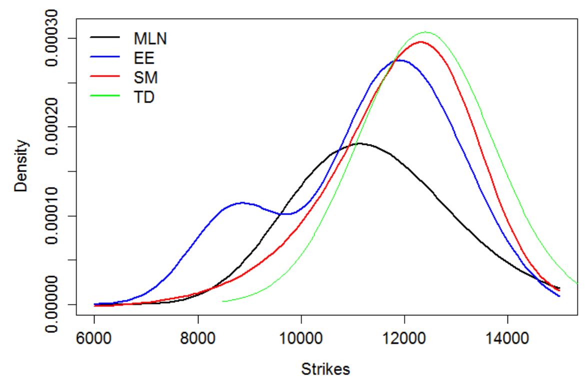
Source: Authors calculation using R Studio and Thomson Reuters data.

Figure 12 Comparison of risk-neutral densities obtained on June 22, 2018 for DAX index towards the true density of DAX index with a maturity date of July 20, 2018



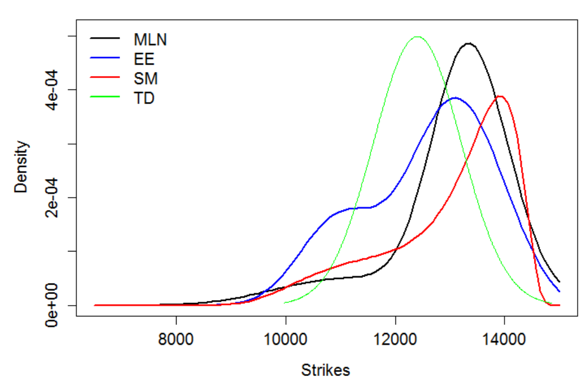
Source: Authors calculation using R Studio and Thomson Reuters data.

Figure 13 Comparison of risk-neutral densities obtained on March 23, 2018 for DAX index towards the true density of DAX index with a maturity date of September 21, 2018



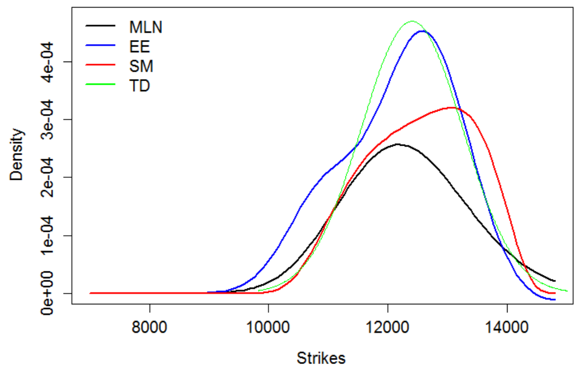
Source: Authors calculation using R Studio and Thomson Reuters data.

Figure 14 Comparison of risk-neutral densities obtained on May 18, 2018 for DAX index towards the true density of DAX index with a maturity date of September 21, 2018



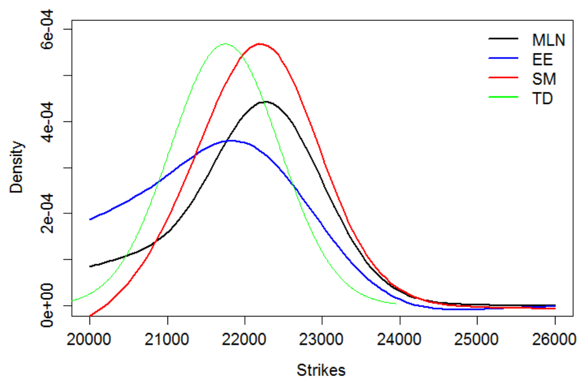
Source: Authors calculation using R Studio and Thomson Reuters data.

Figure 15 Comparison of risk-neutral densities obtained on June 22, 2018 for DAX index towards the true density of DAX index with a maturity date of September 21, 2018



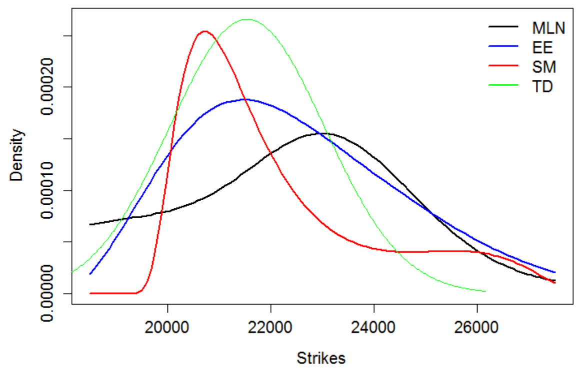
Source: Authors calculation using R Studio and Thomson Reuters data.

Figure 16 Comparison of risk-neutral densities obtained on June 22, 2018 for MIB index towards the true density of MIB index with a maturity date of July 20, 2018



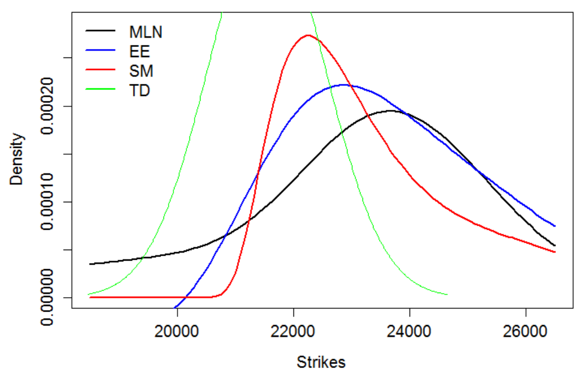
Source: Authors calculation using R Studio and Thomson Reuters data.

Figure 17 Comparison of risk-neutral densities obtained on March 23, 2018 for MIB index towards the true density of MIB index with a maturity date of September 21, 2018



Source: Authors calculation using R Studio and Thomson Reuters data.

Figure 18 Comparison of risk-neutral densities obtained on May 18, 2018 for MIB index towards the true density of MIB index with a maturity date of September 21, 2018



Source: Authors calculation using R Studio and Thomson Reuters data.

IS ROBUST TWO TIMES SCALED ESTIMATOR SUPERIOR AMONG COMPETING ALTERNATIVES OF REALIZED VOLATILITY?

ABSTRACT

We investigate whether the robust two times scaled estimator is superior among alternative estimators of integrated variance by utilizing high-frequency data on a 1 second time scale over a 7-year period. Two groups of estimators are considered, estimators which are robust to microstructure noise and estimators robust to price jumps. The performance of estimators is tested on stock market indices from developed European countries. The analyzed indices are liquid, heavily traded and exhibit intensive intraday activity. The optimal sampling frequency of each estimator is determined with respect to the tradeoff between its bias and the variance and individually adjusted to features of each stock market index. In addition to probability integral transformation test and Mincer-Zarnowitz regression, upper tail dependence from the Gumbel copula is considered as an appropriate pairwise comparison measure. The superiority of robust two times scaled estimator is proven for all the analyzed markets with respect to the optimal slow time scale sampling frequency. Finally, we provide recommendations for researches and practitioners regarding the usage of robust two times scaled estimator for each market separately.

Keywords: high-frequency observations, realized volatility, microstructure noise, price jumps, sampling frequency

JEL classification: C58, D53, G15

Introduction

Estimation and forecasting of variance of price returns is a challenge for researchers and practitioners alike, since financial time series have many features that should be considered in modelling process, such as strong persistence, long memory, asymmetry, non-normality and price jumps. Various approaches can be used, parametric or nonparametric ones, but the main issue is to determine the prediction accuracy of realized variance when a true variance of returns, so called integrated variance (IV) is unknown (Barndorff-Nielsen & Shephard, 2002). IV can be efficiently estimated by the realized variance (RV). It has been shown that the RV converges in probability to IV (Andersen, Bollerslev, Diebold & Labys, 2003). RV is a perfect measure of IV when high-frequency prices are observed in continuous time and without estimation errors. When prices are measured with the presence of microstructure noise, the realized variance may not obtain the same properties like consistency and asymptotic unbiasedness (Hansen & Lunde, 2005). Microstructure noise captures various parts of the trading process: bid–ask bounces, discreteness of price changes, differences in trade sizes or informational content of price changes and strategic component of the order flow.

In practice, stock prices are only observed at discrete time points and high-frequency data is not available for all stocks and markets when trading is not so frequent. Even though the popularity of high-frequency data has risen in the past decade, it is still hard to obtain such data. Along with non-synchronous trading and bid-ask bounce, RV estimator becomes biased, especially when sampling frequency is extremely high, for example 1 second (Bandi & Russel, 2008). In different sampling schemes, where there is no microstructure noise, the realized variance is unbiased (Oomen, 2005). The microstructure noise in intraday data is not suitable for realized variance measure because of the bias problem (Hansen & Lunde, 2005). It has been shown that the high sampled data was contaminated by microstructure noise (Bandi & Russel, 2008). The widespread opinion is not to sample too frequently, i.e. sampling sparsely is recommended in the literature, from 5 min to 30 min (Aït-Sahalia, Mykland & Zhang, 2005). In this context, it is important to choose an appropriate sampling frequency based on a trade-off between accuracy and potential biases (Aït-Sahalia et al., 2005; Hansen & Lunde, 2005). Estimators are biased due to microstructure noise, and in order for them to be unbiased one must find an optimal sampling frequency (Aït-Sahalia, Mykland & Zhang, 2011; Zhang, 2011). When the optimal sampling frequency is found, RV can be used as a measure of IV.

To avoid or reduce microstructural noise, a wide range of robust realized variance estimators emerged (Aït-Sahalia, Mykland & Zhang, 2005; Barndorff-Nielsen & Shephard, 2006; Boudt & Zhang, 2013; Zhang, 2011). Microstructure noise robust estimators are two times scaled realized variance and average subsampled realized variance. This research incorporates pairwise comparison of these estimators, as well as price jump robust estimators to conclude if some are superior to others. Previous studies discuss that daily RV gives volatility forecasts that are prevalent to estimates constructed from AutoRegressive Conditional Heteroskedasticity (ARCH) model proposed by Engle (1982) and its generalized version GARCH model proposed by Bollerslev (1986). These models have been widely used in empirical researches, but they consider only daily close-to-close returns for volatility estimation. While using high-frequency data, the volatility estimators need to be robust to microstructure noise because of its presence in high sampled datasets (Boudt & Zhang, 2013). Therefore, a two times scaled estimator was introduced to overcome the microstructure noise when there are no price jumps (Zhang, Mykland & Aït-Sahalia, 2005). When jumps occur, a two times scaled estimator becomes biased (Zhang et al., 2005). In the previous literature many jump robust estimators were established, but most of them are not robust to microstructure noise (Barndorff-Nielsen & Shephard, 2006; Boudt, Croux & Laurent, 2011). Therefore, the robust two times scaled estimator is presented with detailed analysis that empirically supports its robustness to both microstructure noise and price jumps. For the same reason, the robust two times scaled estimator is used as a benchmark in this research. In the existing literature, performance of estimators that are robust to microstructural noise, price jumps or both have been proven based on simulations or data from the US market, while there is no research at all that analyzes the developed European stock markets (Floros et al., 2020). In the previous literature there have been empirical as well as simulation studies that have described which volatility estimator is appropriate and in which conditions (Andersen et al., 2012; Hanousek, Kočenda & Novotný, 2012). Market microstructure noise that is present in high-frequency data can be overcome by using a two times scaled estimator that is shown to be robust to it (Aït-Sahalia et al., 2011). The most common estimators that are robust only to price jumps are minimized block of two returns and medianized block of three returns measures, and they are shown to be appropriate in usage when in a setting where price jumps are substantial (Andersen, Dobrev & Schaumburg, 2012). When there are significant price jumps in high-frequency data it is shown that the appropriate measure to use is bipower variation (Andersen, Bollerslev & Diebold, 2007; Corsi, Pirino & Reno, 2010). In the presence of microstructure noise and price jumps, the robust two times scaled estimator remains unbiased (Zhang et al., 2005; Boudt & Zhang, 2013). While estimating the integrated variance in a simulation study, looking at the asymptotic properties, the two times scaled estimator was shown to be consistent and unbiased to microstructure noise (McAleer & Medeiros, 2008). In an extensive simulation study, it has been shown that the best estimator for price jumps is the one based on bipower variation (Hanousek, Kočenda & Novotný, 2012). In previous studies there is no consensus on which estimator is better for which observed market. This research provides an answer to that question. Used methodologies serve for comparison purposes and give the direction for concluding on the main objective. In addition, the superiority of the robust two times scaled estimator is shown by comparison with two groups of estimators by simultaneously discovering which observed developed European markets are more contaminated with price jumps and which contain more microstructure noise. The main objective of this paper is to determine whether the robust two times scaled estimator is superior to alternative volatility estimators for four developed European markets observed. This research employs the optimal sampling frequency at slow time scale for the observed financial markets and thus it is approached correctly in order to solve the main objective. This is extremely important not only for investors, but also for policy makers and risk managers.

The outline of the paper is as follows: in Section 2 the used methodology and data are presented, Section 3 gives empirical results of the tests performed, while conclusions are provided in Section 4.

Data and methodology

In this paper high-frequency data of developed stock markets is considered, covering the period from January 4, 2010 to April 28, 2017 for Germany, Italy, France and UK. The data was provided by Thomson Reuters Tick History. The fast time scale sampling frequency of 1 second is determined in advance, according to data availability of the observed financial markets within the shortest, nonempty and equidistant intervals. In order to define the optimal slow time scale sampling frequency, the root mean square error (RMSE) of the proposed estimator is used. The RMSE of each estimator was calculated as the sum of its squared bias and its variance, and afterwards the RMSE was minimized with respect to slow time scale frequency which corresponds to the number of subsamples.

The descriptive statistics of the data including the optimal sampling frequency are presented in Table 1.

Table 1 Description of high-frequency data from January 4, 2010 to April 28, 2017

European market	Stock index	No. of trading days	No. of 1 sec. observations	Optimal slow time scale
Italy	MIB	1862	42773317	10 sec.
Germany	DAX	1863	56650069	20 sec.
France	CAC	1878	4665980	13 sec.
UK	FTSE	1850	48920222	30 sec.

Source: Authors based on Thomson Reuters data

In Table 1 the number of trading days and number of 1-second observations are given. These numbers vary depending on the observed European market. The intraday data taken into consideration was during trading hours from 9 a.m. until 5:30 p.m. for all developed European markets.

Table 2 Realized volatility estimators

IV estimator	Formulation
Realized variance	$RV_t^{\Delta} = \sum_{i=1}^{n_t} r_{t_i}^2$
Bipower variation	$BPV_t^{\Delta} = \frac{\pi}{2} \sum_{i=2}^{n_t} \lvert r_{t_{i-1}} \rvert \lvert r_{t_i} \rvert$
Minimized block of two returns	$MinRV_t^{\Delta} = \frac{\pi}{\pi - 2} \frac{n_t}{n_t - 1} \sum_{i=2}^{n_t} m\,in\left(\lvert r_{t_{i-1}} \rvert, \lvert r_{t_i} \rvert\right)^2$
Medianized block of three returns	$MedRV_t^{\Delta} = \frac{\pi}{6 - 4\sqrt{3} + \pi} \frac{n_t}{n_t - 2} \sum_{i=2}^{n_t - 1} m\,edian\left(\lvert r_{t_{i-1}} \rvert, \lvert r_{t_i} \rvert, \lvert r_{t_{i+1}} \rvert\right)^2$

Average subsampled realized variance	$ARV_t^{\Delta,k} = (\frac{n_t}{n_t - k + 1}) \frac{1}{k} \sum_{j=1}^{k} \sum_{i=1}^{n_t} r_{t_{ij}}^2$
Two times scaled realized variance	$TSRV_t^{\Delta,k} = (1 - \frac{n_t - k + 1}{n_t k})^{-1} (\frac{1}{k} \sum_{j=1}^{k} \sum_{i=1}^{n_t} r_{t_{ij}}^2 - \frac{n_t - k + 1}{n_t k} \sum_{i=1}^{n_t} r_{t_i}^2)$
Robust two times scaled realized variance	$RTSRV_t^{\Delta,k,\theta} = (1 - \frac{n_t - k + 1}{n_t k})^{-1} c_\theta (\frac{1}{k} \sum_{j=1}^{k} \sum_{i=1}^{n_t} r_{t_{ij}}^2 I_i(\theta) - \frac{n_t - k + 1}{n_t k} \sum_{i=1}^{n_t} r_{t_i}^2 I_i(\theta))$
Hayashi-Yoshida realized variance	$HYRV_t^{\Delta} = \sum_{i=1}^{n_t} r_{t_i}^2 (I^i)$

Source: Authors

Eight estimators of integrated variance (IV) are presented in Table 2. Each of them depends on sampling frequency Δ, i.e. the increment between two successive and equally spaced observations $\Delta = t_i - t_{i-1}$. The number of intraday returns r_{t_i} for every day is n_t. Since $n_t = 1/\Delta$ the lesser number of returns is used when Δ increases. Moreover, realized variance RV_t^{Δ} becomes unbiased when sampling frequency Δ increases due to reduction of microstructure noise as a consequence of sparse sampling (Andersen & Bollerslev, 1998). However, the cost of sparse sampling is a high variance of an estimator (inconsistency) as a small number of observations is left for computational reasons. Not only the microstruture noise contaminates realized variance, but also price jumps. Thus, a second estimator BPV_t^{Δ} is proposed in the literature as jumps robust estimator. The idea of Barndorff-Nielsen & Shephard (2006), who introduced a bipower variation, is that multiplication of $|r_{t_i}|$ with the adjacent $|r_{t_{i-1}}|$ will dampen the jump impact if it occurs even for sufficiently small Δ. Similar to bipower variations, two estimators also emerged in the literature, i.e. minimized and medianized realized variance $MinRV_t^{\Delta}$ and $MedRV_t^{\Delta}$, respectively. According to Andersen et al. (2012), both estimators follow a concept of the nearest neighbor truncation by the use of the minimum operator on blocks of two returns and the median operator on blocks of three returns. They have better performance in the finite samples compared to bipower variation, and even $MinRV_t^{\Delta}$ suffers from a similar exposure to zero returns as BPV_t^{Δ}. A serious drawback of the aforementioned estimators is the loss of data when sampling sparsely. Opposite to that, Zhang et al. (2005) proposed how to keep all the data but still have an unbiased and asymptotically consistent estimator of IV. Thus, a two time scaled estimator $TSRV_t^{\Delta,k}$ was introduced, which combines average subsampled realized variance at slow time scale $ARV_t^{\Delta,k}$ and realized variance RV_t^{Δ} at fast time scale. In other words, when utilizing the two time scale estimator, parameter Δ is the fast time scale, i.e. the highest possible sampling frequency available to the user, while the parameter k is the slow time scale frequency which defines the number of subgrids. For practical reasons, it is common to fix the fast time scale while the slow time scale should be determined optimally by minimizing RMSE (root mean square error) of the estimator. As previously highlighted, RV_t^{Δ} is robust to microstructure noise as well as $ARV_t^{\Delta,k}$ and $TSRV_t^{\Delta,k}$ depending on the parameters Δ and k, while BPV_t^{Δ}, $MinRV_t^{\Delta}$ and $MedRV_t^{\Delta}$ are robust to price jumps only. Robust estimators of IV have also been designed in the presence of both jumps and noise. In particular, robust version of two times scaled realized variance $RTSRV_t^{\Delta,k,\theta}$ was designed by Boudt & Zhang (2013). This estimator requires one additional parameter θ which is employed as a threshold with respect to indicator function $I_i(\theta)$. The threshold is usually set to 9 indicating if returns are larger than three standard deviations from the mean. If returns are larger than three standard deviations from the mean, the indicator function has value 0 and 1 otherwise. The last estimator which is considered in this paper for comparison is $HYRV_t^{\Delta}$ designed by Hayashi & Yoshida (2005). It can be understood as a threshold realized variance which uses jump detection rule I^i to truncate the effect of jumps. It has to be mentioned that all estimators are sensitive to selection of sampling frequency, and criteria for optimal frequencies should be considered in the presence of noise and jumps. Comprehensive study with respect to these criteria can be found in Bandi & Russel (2008) and Aït-Sahalia et al. (2005).

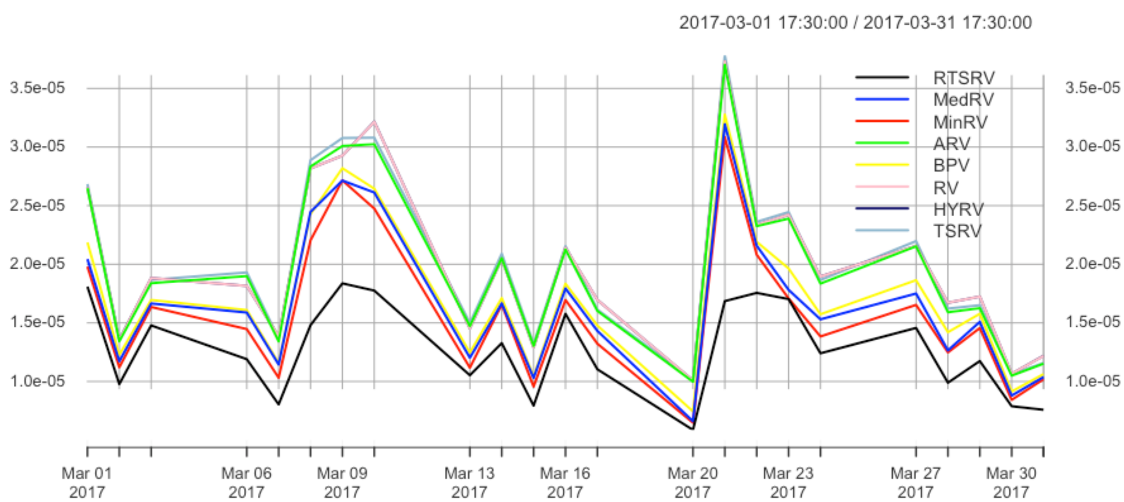


Figure 1 All realized volatility estimators for German stock market

Figure *1* presents eight realized volatility estimators for the German stock market index (DAX). For better visualization of the difference between estimators, the data frame used was from March

1, 2017 until March 31, 2017. After the calculation of realized volatility estimators and $RTSRV_t^{\Delta,k,\theta}$, three comparison methods are conducted in order to determine which volatility estimator fits best with the benchmark. These three comparison methods are Mincer-Zarnowitz regression, probability integral transformation test and Gumbel copula upper tail dependence. Mincer-Zarnowitz regression is based on the overall performance of the realized volatility models. PIT test was performed as a density goodness of fit procedure in order to test how the examined realized volatility estimators preform in comparison to $RTSRV_t^{\Delta,k,\theta}$. The upper tail dependence was used because it examines what happens in the extreme values or tails.

The $RTSRV_t^{\Delta,k,\theta}$ is defined as a benchmark because it is confirmed that $RTSRV_t^{\Delta,k,\theta}$ is robust to market microstructure noise, jumps and non-synchronous trading in the intraday stock price series. This advantage from other realized volatility estimators was verified by the reduced bias and mean square error in a simulation study (Boudt & Zhang, 2013).

Empirical results

Each comparison method comprises fitting seven realized volatility estimators to the benchmark that is robust two times scaled estimator ($RTSRV_t^{\Delta,k,\theta}$). The optimal sampling frequency was selected for each European market, based on minimizing the root mean squared error (RMSE) (Ait-Sahalia et al., 2005; Arnerić, Matković & Sorić, 2019). That is when the $RTSRV_t^{\Delta,k,\theta}$ becomes unbiased to microstructure noise. For the calculation of the RV, it is suggested to use returns that are sampled as often as possible because we get the maximum amount of information from data that is ultra-high-frequency. However, if the sampling frequency is as high as it can be, it leads to a bias problem due to microstructure noise (Oomen, 2005). There is a trade-off between the bias and efficiency while determining the sampling frequency. One must establish the optimal sampling frequency for an observed financial market in order to reduce the bias but for a volatility estimator to still remain efficient (Ait-Sahalia et al., 2005; Bandi & Russel, 2008). In the presence of jumps there will also be bias in practical applications of estimators that are not jump robust. So it will have an effect on sampling frequency which is also influenced by market structure, liquidity and microstructure noise. The way to increase the efficiency of estimators is to sub-sample (taking the average of an estimator across all possible sub-samples) (Aït -Sahalia et al., 2005; Andersen et al., 2012). With microstructure robust estimators the optimal sampling frequency is obtained by minimising the mean square error (MSE) (Arnerić et al., 2019; Zhang, 2011). As seen in Figure *2,* the optimal sampling frequency was established for each European market. For Germany it is 20 seconds, for UK it is 30 seconds, for Italy it is 10 seconds and for France it is 13 seconds. It shows the root mean squared error (RMSE) of the robust two times scaled estimator $RTSRV_t^{\Delta,k,\theta}$ for each observed developed European market index against the number of subsamples. By minimising the RMSE against the number of subsamples, the optimal sampling frequency (optimal slow time scale) is obtained.

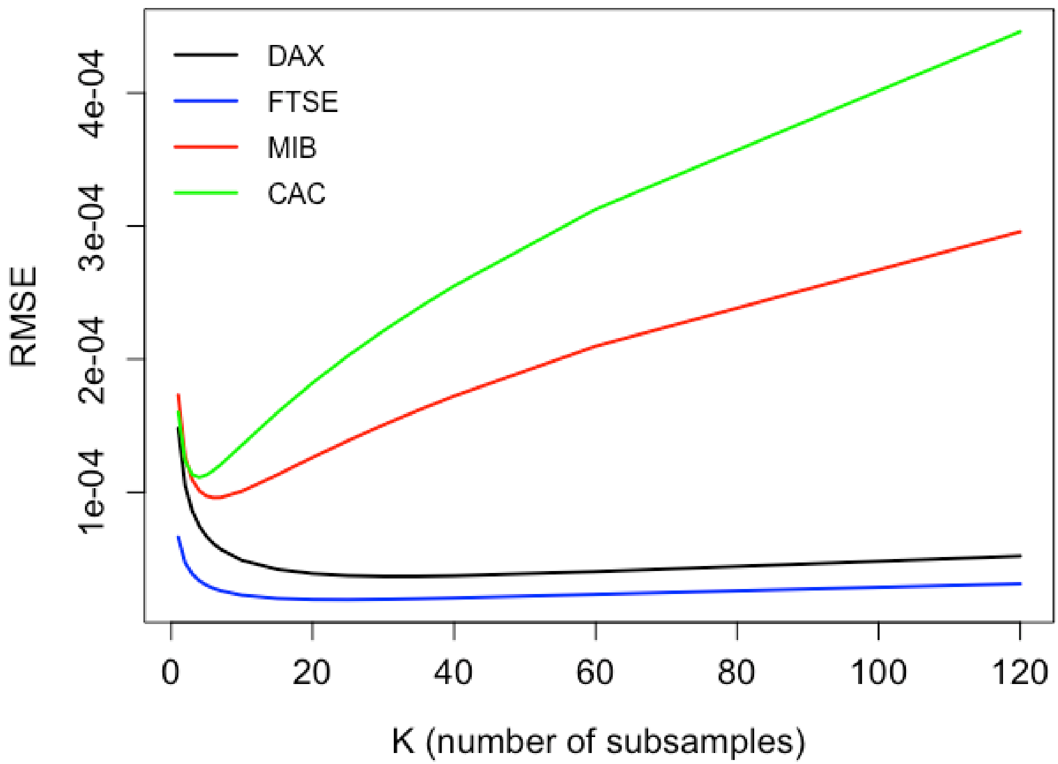


Figure 2 Root mean squared error of the RTSRV estimator for each European market index against the number of subsamples

The comparison methods used were Mincer-Zarnowitz regression, probability integral transformation (PIT) test and Gumbel copula upper tail dependence. The results of the first two methods for comparison of the realized volatility estimators indicate that RV_t^{Δ}, $TSRV_t^{\Delta,k}$, $ARV_t^{\Delta,k}$ and $HYRV_t^{\Delta}$ underestimate the performance of estimates during severe stress and price jumps. Therefore, the results do not give a clear answer which estimator fits the best to the benchmark. In that case, the upper tail dependence is a favorable method to use because it takes into account extreme values. Each competing volatility estimator was tested against $RTSRV_t^{\Delta,k,\theta}$ using Mincer Zarnowitz regression, PIT test and upper tail dependence measure within the Gumbel copula. Firstly, the Mincer-Zarnowitz regression was used where it was tested how well the volatility estimators fit to $RTSRV_t^{\Delta,k,\theta}$:

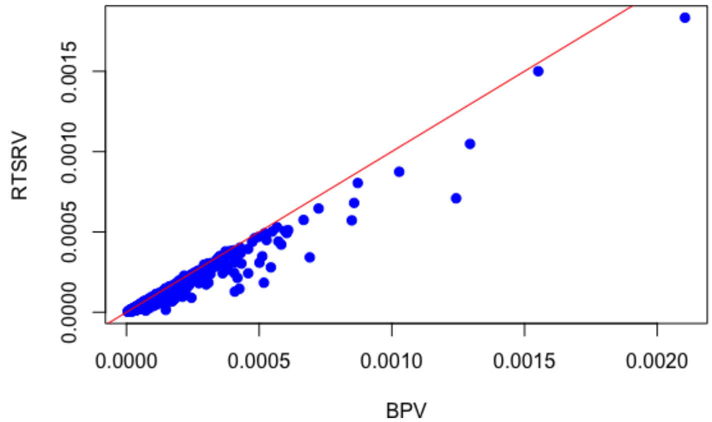
$$\hat{\sigma}^2_{RTSRV} = \beta_0 + \beta_1 \hat{\sigma}_t^2 + \epsilon_t$$

More specifically, it was tested whether $\beta_0 = 0$ and $\beta_1 = 1$. The results, including chi-squared test statistic and p-value, are presented in Table 3 below. For four European markets and seven volatility estimators the null hypothesis was rejected at significance level of 5%. This showed how seven observed volatility estimators do not fit well to $RTSRV_t^{\Delta,k,\theta}$. Figures $3.a$ and $3.b$ are scatter plots showing the positive relationship between $RTSRV_t^{\Delta,k,\theta}$ and two other realized volatility estimators. They present how well the bipower variation (BPV_t^{Δ}) and average subsampled realized variance ($ARV_t^{\Delta,k}$) fit to robust two times scaled realized variance. Scatter plot indicates that both bipower variation (BPV_t^{Δ}) and average subsampled realized variance ($ARV_t^{\Delta,k}$) underestimate the variance i.e. benchmark.

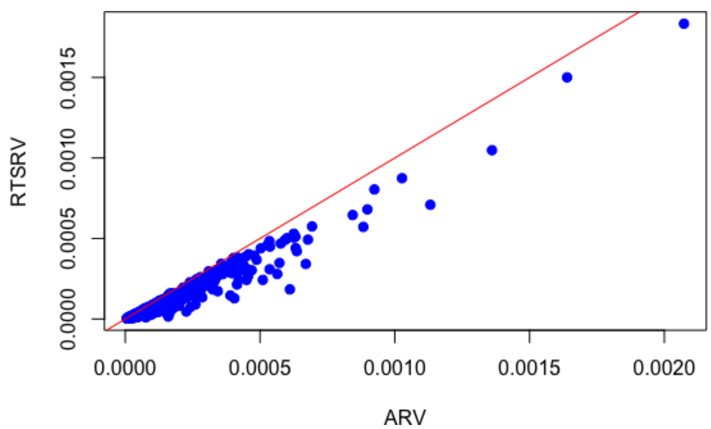
Table 3 Results of Mincer-Zarnowitz regression

	$MedRV_t^{\Delta}$	$MinRV_t^{\Delta}$	RV_t^{Δ}	BPV_t^{Δ}	$TSRV_t^{\Delta,k}$	$ARV_t^{\Delta,k}$	$HYRV_t^{\Delta}$
MIB							
test statistic	3449.3***	2298***	8375.1***	5442.6***	8841.3***	7530***	8274***
β_0	0.000002	0.000002	0.000008	0.000001	0.000007	0.000007	0.000008
β_1	0.779631	0.799547	0.508353	0.739918	0.504905	0.518399	0.508959
DAX							
test statistic	2281.7***	1853.3***	6517.3***	3404.7***	8001.1***	6732.8***	6517.1***
β_0	-0.000002	-0.000001	-0.000008	-0.000003	-0.000009	-0.000009	-0.000008
β_1	0.850240	0.856405	0.766997	0.820539	0.763453	0.782201	0.766998
CAC							
test statistic	2359.3***	3739***	4611.3***	2776.1***	1853.2***	679.25***	470.67***
β_0	0.000010	0.000011	0.000004	0.000008	0.000002	0.000005	0.000005
β_1	1.275828	1.426721	1.006286	1.355853	0.792073	1.012190	1.006807
FTSE							
test statistic	4296***	4039.8***	7845.9***	5365.1***	9819.5***	8446.9***	7845.7***
β_0	0.000001	0.000001	-0.000002	-0.000001	-0.000002	-0.000002	-0.000003
β_1	0.782853	0.777687	0.724623	0.769927	0.713995	0.730009	0.724622

Note: ***, **, * represent significance of the chi-squared test at the 1%, 5% and 10% level.



(a) Bipower variation



(b) Average subsampled realized variance

Figure 3 Relationship between robust two times scaled realized variance and other realized volatility estimators

The probability integral transformation (PIT) test was used to check whether the difference between $RTSRV_t^{\Delta,k,\theta}$ and other competing volatility estimators is uniformly distributed. The results, including Kolmogorov-Smirnov test statistic for uniformity and p-value, are presented in Table 4 below. For all European markets and all volatility estimators the null hypothesis was rejected at significance level of 5%.

Table 4 Results of PIT test

	$MedRV_t^{\Delta}$	$MinRV_t^{\Delta}$	RV_t^{Δ}	BPV_t^{Δ}	$TSRV_t^{\Delta,k}$	$ARV_t^{\Delta,k}$	$HYRV_t^{\Delta}$
MIB							
test statistic	14399.1***	16639.3***	9577.5***	14373.9***	8801.2***	10161.2***	9554.1***
DAX							
test statistic	15123.4***	15272.9***	12953.1**	16349.2**	10790.7***	11598.8***	12955***
CAC							
test statistic	10636**	10152***	11580**	10692***	17964***	11624***	11579***
FTSE							
test statistic	20727**	12495***	13824**	18095***	14016***	13099***	13824***

Note: ***, **, * represent significance of the Kolmogorov-Smirnov test at the 1%, 5% and 10% level.

This research utilizes the upper tail dependence coefficient, a result of the Gumbel copula function. When the focus of interest is the upper tail of the distribution, it is used for comparison purposes. While the Mincer-Zarnowitz regression and PIT test haven't shown the preference of a specific estimator, the upper tail dependence is utilized. Representing the extreme value distributions, the Gumbel copula function is an upper tail dependence measure given by:

$$C(u,v) = \exp\left\{-\left[-\ln(u)^{\delta} + \left(-\ln(v)^{\delta}\right)\right]^{\frac{1}{\delta}}\right\}$$

where $u, v \in [0,1]$, with parameter $1 \leq \delta \leq \infty$ that controls the strength of reliance. Upper tail dependence is a function of Gumbel copula parameter $\lambda_u = 2 - 2^{\frac{1}{\delta}}$.

Table 5 Upper tail dependence results

	$MedRV_t^{\Delta}$	$MinRV_t^{\Delta}$	RV_t^{Δ}	BPV_t^{Δ}	$TSRV_t^{\Delta,k}$	$ARV_t^{\Delta,k}$	$HYRV_t^{\Delta}$
MIB							
δ	7.09	6.81	5.35	6.72	5.23	5.51	5.37
λ	0.859	0.853	0.813	0.851	0.809	0.819	0.814
DAX							
δ	7.96	7.65	6.51	7.6	6.64	6.65	6.51
λ	0.874	0.869	0.846	0.868	0.849	0.85	0.846
CAC							
δ	5.86	5.72	5.75	5.93	6.93	5.85	5.75
λ	0.829	0.825	0.826	0.831	0.856	0.829	0.826
FTSE							
δ	7.34	7.02	5.69	7.06	5.81	5.8	5.69
λ	0.864	0.858	0.824	0.858	0.828	0.828	0.824

Source: Authors' calculation using R Studio

The results given in Table 5 present the tail dependence coefficient λ based on the Gumbel copula function and δ that regulates the degree of reliance. The results indicate that for Italy, Germany and UK, $RTSRV_t^{\Delta,k,\theta}$ has the highest upper tail dependence with $MedRV_t^{\Delta}$, $MinRV_t^{\Delta}$ and BPV_t^{Δ} volatility estimators. Among all the competing estimators, only jump robust ones have produced almost similar volatility estimates as $RTSRV_t^{\Delta,k,\theta}$ (Andersen et al., 2012; Barndorff-Nielsen & Shephard, 2006). In case of France, $RTSRV_t^{\Delta,k,\theta}$ is best fitted with $TSRV_t^{\Delta,k}$, BPV_t^{Δ}, $ARV_t^{\Delta,k}$ and $MedRV_t^{\Delta}$ volatility estimators. As $TSRV_t^{\Delta,k}$ is robust to microstructure noise, it is of no surprise that estimates are as good as $RTSRV_t^{\Delta,k,\theta}$.

Conclusion

Over the past decade, the scientific focus on research of volatility estimators has increased and expanded the knowledge from the already existing literature. This increase is mainly due to the availability of high-frequency data. This gave light to the as of now existing theory of realized volatility, which indicated that the usage of intraday data is needed for more precise volatility measures. Even though realized variance (RV_t^{Δ}) is the most used high-frequency estimator, it is biased due to microstructure noise. The benchmark robust two times scaled realized variance ($RTSRV_t^{\Delta,k,\theta}$) is microstructure noise and jump robust. In this research the data observed is intraday 1 second observations. It is determined that the optimal sampling frequency for the robust two times scaled realized variance ($RTSRV_t^{\Delta,k,\theta}$) is from 10 to 30 seconds. There is no consensus in the previous studies regarding the "best" realized volatility estimator. The objective of this research paper is to determine whether the robust two times scaled realized variance ($RTSRV_t^{\Delta,k,\theta}$) is a superior volatility estimator for each of the four considered European markets by performance comparison of two groups of estimators, i.e. estimators which are robust to microstructure noise as well as jump-robust estimators. Due to inconclusive results from Mincer-Zarnowitz and PIT test, the upper tail dependence was introduced because it examines the events in tails i.e. extreme values. The results indicated that the medianized block of three returns ($MedRV_t^{\Delta}$) performed most similar to the robust two times scaled realized variance ($RTSRV_t^{\Delta,k,\theta}$) for Italy, Germany and UK. For France the two times scaled realized variance ($TSRV_t^{\Delta,k}$) realized volatility estimator was the most similar (approximately equal) to the benchmark. Since medianized block of three returns ($MedRV_t^{\Delta}$) is robust only to price jumps, we conclude that the Italian, German and UK financial markets are more contaminated by price jumps than by microstructure noise at selected sampling frequencies. The French financial market is more contaminated by microstructure noise than by price jumps. This contributes to the existing literature in several ways. The main finding considers the selection of optimal slow time scale frequency in favor of two times scaled estimator in each market individually, at the same time ensuring robustness to price jumps. This research contributes to the previous studies with an empirical dataset consisting of high-frequency price observations comprising four main European market indices (DAX, CAC, FTSE and MIB), because there are very few studies that take into account the calculations of realized volatility estimators on developed European markets. Another novelty is the usage of a combination of three tests for benchmarking: Mincer-Zarnowitz regression, PIT test and upper tail dependence test within the Gumbel copula where the results are given for each of the observed developed European markets. The results are important to financial analysts and investors because they offer a recommendation which realized volatility estimator to use for the observed market indices. An additional contribution is also a determined optimal sampling frequency for each of the observed developed European markets. The limitation we encountered was in data collection, because they are not easily accessible. The next steps would be to investigate the realized covariance of different assets. The correlation is a standardized covariance and it is sensitive to sampling frequency. The direction of further research would be to examine the correlation among traditional assets. In this way, the characteristics of bonds and commodities as separate assets would also be explored. Therefore, it would be useful to see the relationship between various asset examples like bonds and stock indices.

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BENEFITS OF SECTORAL CRYPTOCURRENCY PORTFOLIO OPTIMIZATION

Abstract

When creating a portfolio, investor should consider the dynamics of the income ratio of the portfolio asset selected in order to identify and quantify the taken risk of the investment. This research paper will formally identify and describe the benefits of sectoral cryptocurrency classification portfolio optimization and it's performance. Six optimization targets will be formed: MinVar, MinCVaR, MaxSR, MaxSTARR, MaxUT and MaxMean. We compare the obtained portfolios with the performance of the CRIX index (representing the crypto market) over the same period. Our results show that five of the six portfolio strategies performed better if they included sectoral cryptocurrencies namely from financial, exchange and business services sectors.

Keywords: cryptocurrency, portfolio optimization, sectoral classification, investments
JEL classification: E49, G11, P45

1. Introduction

Continuous development of information technology and computing power, as well as an increase of distrust in the global financial and payment system in response to the recent economic crises and geopolitical tensions, is intensifying work on the existing ideas of digital currencies and assets. As a result, on October 31, 2008, a document entitled "Bitcoin: A Peer-to-Peer Electronic Cash System" describes a new decentralized transaction system that does not include an intermediator between entities of interest. The first description of the article's summary states a person signing up as Satoshi Nakamoto on the Internet at http://www.metzdowd.com along with a link to the entire article of the same author. Already at the outset of the system characteristics described by the author, one can see how revolutionary, far-reaching and promising the idea of his project was. The computer program, or the Bitcoin protocol, was released to the public on January 9, 2009, creating the world's first cryptocurrency - bitcoin. Its technology enabled almost instantaneous transaction execution, with negligible fees without intermediaries or central body, which attracted great attention as well as a large number of users themselves. An important characteristic of the Bitcoin protocol is its open source system, the system whose initial code is open and free to the public. This means that anyone who has an interest can freely study and work on the system - if their suggestions are in the direction of improvement, the community will accept changes and improve the protocol. Again, this also means that anyone who wants can freely take the existing protocol, modify it, in some context change or adapt to their needs, create a new cryptocurrency and release it to the public. The latter allowed the creation of a number of new cryptocurrencies with different properties and spreading their use first in payment transactions and then in the context of their trading on the new secondary market.

2. An overview of current research

At its core, the Bitcoin platform is a decentralized transaction system, that is, a distributed ledger, secured by cryptography and managed by consensus, which records all transactions that occurred between participants in the community. The general ledger consists of transactions grouped into so-called blocks from which the ledger is actually a blockchain. At the end of each block is a summary of that block, that is, a summary of all previous transactions written as a syntax of the hash or digest function (Härdle et al., 2019). This record is distributed throughout the network and represents the record for the next block of transactions. In addition to summarizing all transactions, the network also distributes a new block of transactions. Since the digest function is a one-way mathematical algorithm that has the same input data converts to an output record with a unique structure, if there is a discrepancy in the result of the nodes function, this would mean that there has been a change in data, either in previous transactions, or in a new transaction block created. In other words, one of the participants in the network has changed the balance sheet of one of the accounts in the network, and such a block of transactions is rejected and classified as incorrect. The process just described is a solution to a problem that has long been a stumbling block in the context of the development of digital currencies, which is double expense.

Digital record and blockchain technology allows transparency because every transaction record is visible and publicly available. With the implementation of such technology, the consumption of each unit of money would be public, thus completely eliminating the possibility of malpractice, corruption, etc. Except in the context of the value of units that can be expressed up to 8 decimal places (Symitsi and Chalvatzis, 2018), the application of such technology is very wide. For example, accounting information systems are already being developed based on a distributed ledger because it is actually an electronic record that can serve a variety of purposes, such as a record of balances on accounts of customers and suppliers, proof of ownership of financial instruments, music record, etc. The technological advances and practical applications of blockchain-based cryptocurrencies can also be seen as a new type of digital asset (Glaser et al., 2014). There are different categorizations and definitions of cryptocurrencies, however for the time being none of them is fully accepted or there is a consensus as to what type of existing assets cryptocurrencies represent.

Although the design of the cryptocurrency market in its initial phase was based solely on the parameterization of the existing Bitcoin protocol (Elendner et al., 2016), it is the open source feature and practical implementation of blockchain technology - characteristics that have been recognized by young and innovative companies to raise the capital needed for their development on the one hand, and the positive public reaction to the idea of decentralization, on the other.

In this paper, we investigate relationship among cryptocurrencies and cryptocurrency sectors with the aim of constructing and modeling superior portfolios, in the sense that such portfolios can beat the market. Fundamentally, investors apply different techniques, models and strategies to construct their own portfolio whose performance dynamics should outperform the market, that is, a portfolio that should yield more than market equilibrium returns. Such a definition entails the pursuit of undervalued assets, which would ultimately result in a market that is information-efficient, that is, a market whose aggregate value reflects all relevant and available information related to individual assets. If the standard definitions of investing are placed in the context of the cryptocurrency market, serious discrepancy may be noted. The first thing that cannot be put in the context of the cryptocurrency market is the syntagma market in equilibrium. For a market to be in equilibrium, it would mean that there is a market consensus on the expected rate of return on the assets being invested. For there to be market consensus, it implies the existence of its fundamental value, because how to create a consensus on the expected return on an asset if two independent investors value the same assets differently. Secondly, for the market to be information-efficient - it is necessary to first define which news (information) affect the price of cryptocurrencies, and only then examine whether there is a positive or negative price response to the news (e.g., forking Bitcoin system which means that the number of existing bitcoin units is duplicated and the value of bitcoin remains the same as before the release of that information). Therefore, due to the absence of at least approximately equal valuation, undervalued or overvalued assets do not exist, and therefore there is neither market consensus on expected rates of return nor a market in equilibrium that is information efficient. However, despite all of the above, the cryptocurrency market and its entire infrastructure is continuously growing year on year. Due to its availability, an increasing number of institutional and individual investors of different profiles invest and trade in cryptocurrencies, slowly making cryptocurrencies a legitimate asset class in investor's view. Since cryptos are entering a mainstage the need for serious financial analysis and continued research is increasing.

Among the first studies on this topic is conducted by (Trimborn, 2015), who in his work optimizes the cryptocurrency portfolio of constituents of the CRyptocurrency IndeX - CRIX index with the aim of minimizing variance. The author faces limitations of missing data and insufficiently of long time series. For the first constraint, a bootstrapping parameter method is used to estimate missing values, and for the second, it applies the General Autoregressive Conditional Heteroskedasticity (GARCH) model while estimating the expected volatility of time series. From a volatility point of view, the results of the approximated data portfolio favor an optimized portfolio where volatility is lower than CRIX. On the other hand, excluding the estimated data, the results favor the CRIX index where volatility is lower and cumulative return is higher.

The cryptocurrency market can be viewed on its own but also in combination with traditional financial instruments and assets. (Trimborn et al., 2018) conducts research into existing cryptocurrency portfolios with an individual market capitalization of more than 1 million dollars and incorporates them with traditional instruments - stocks, components of the S\&P 100 and DAX30 indexes, shares listed on the Portuguese stock exchange, and runs a minimal variance optimization. In order to avoid cryptocurrency liquidity problems they approximate the liquidity measure and create a cap on the upper limit on the allocation of each cryptocurrency in the Liquidity Bounded Risk-return Optimization - LIBRO portfolio. Optimization is carried out with and without limitation on units and the performance of the portfolio is compared. In both cases, the results are in favor of cryptocurrency. Including cryptocurrencies in the portfolio improves the reward-risk ratio. Equity-constrained portfolios of equities and cryptocurrencies produce better cumulative returns than non-restricted portfolios.

(Trimborn et al., 2018) extends previous research and introduces Barclays Capital US Aggregate Index and Commodity Market Index (S&P GSCI) into the analysis. In addition to the standard optimization mean-variance model, it introduces Conditional Value at Risk (CVaR) as a measure of risk. All created portfolios that include cryptocurrencies in their composition, with or without LIBRO equity limitation, have performed better than portfolios created only from traditional assets. In addition, certain portfolios that take CVaR as a measure of risk, appear to have a higher cumulative yield than the standard MV model.

One of the most comprehensive studies examining the performance of a portfolio created from cryptocurrency and traditional assets is conducted by (Petukhina et al., 2018). The authors group the existing standard and recent optimization models into four strategies: risk-oriented strategies, return-oriented strategies, risk-return-oriented strategies and combination strategies. The selected models are applied by the authors to portfolios composed of 55 selected cryptocurrencies and 16 variables represented by 5 types of traditional assets. The LIBRO methodology was also included in the research and portfolios were created with and without equity restrictions to control liquidity risk. The performance of all the portfolios created indicates the usefulness of including cryptocurrencies in a portfolio along with traditional assets. The same portfolios achieved a lower cumulative return in case when the limits on the units controlling the liquidity were raised.

(Lee Kuo Chuen et al., 2018) models market sentiment as the average return of a historical return series and creates a portfolio strategy based on a performed sentiment analysis. The authors optimized a portfolio of ten selected cryptocurrencies along with traditional assets consisting of stock indices, real estate market index and gold. Due to the absence of a normal return distribution, apart from the standard MV model, the authors use CVaR as a measure of risk and compare the performance and allocation of portfolio assets. As in aforementioned research, the inclusion of cryptocurrencies in the portfolio raises the effective limit of possible portfolios, thereby improving the reward-risk ratio. In addition, the strategy created on sentiment analysis has achieved a far higher cumulative return than comparative portfolios, thus confirming the significant sentiment dynamics in the cryptocurrency market.

In previous recent research, the focus has been on a portfolio that includes multiple cryptocurrencies. Since bitcoin is the first cryptocurrency to have some form of secondary market

(Mt. Gox started operating in 2010), and given the rise in its price back in 2013 and 2014, bitcoin has already attracted interest of the scientific community, and is being considered as an individual alternative asset in the area of portfolio modeling. The first papers examining its contribution by including it in a well-diversified portfolio of traditional assets are conducted by (Briere et al., 2015) and (Eisl et al., 2015). In the Briere et al. (2015) paper, the analysis is carried out in the sample, that is, by applying the standard MV model the efficient limit of possible portfolios on the full data sample is derived and its contribution to performance i.e. portfolio diversification is interpreted. Considering the results of the Briere et al. (2015) - the absence of a normal return distribution, in the Eisl et al. (2015) paper an out of sample analysis is performed where CVaR is taken as a measure of risk during optimization. Research results from both papers indicate that bitcoin should be included in the portfolios since - the higher risk is compensate by the higher expected return on the portfolio.

Bitcoin's role in the dynamics of portfolios created from traditional assets by continental affiliation (EU, US and China) is examined by (Kajtazi and Moro, 2018). CVaR was used as a measure of risk with the optimization goal of maximizing portfolio return, except for the portfolio with equal allocations. They conclude that the inclusion of bitcoin cryptocurrency in a well-diversified portfolio of traditional assets contributes to improving the risk-reward ratio. Also, the inclusion of BTC as an asset also generates a higher cumulative return.

(Carpenter, 2016) also examines the impact of including BTC in a well-diversified portfolio of traditional assets, however, using the capital asset pricing model (CAPM) to estimate the expected returns of individual assets in the portfolio. Because the implementation of CAPM requires a statistically significant relationship between the additional returns on the index and the total market return, and the relationship fails to prove the relationship between BTC and the observed US capital market index, the author uses the mean of historical BTC returns for expected returns. Also, in the case of abnormally high BTC returns over the period considered, such as in 2014, the return magnitude is adjusted and reduced to a lower value to eliminate extreme values. The analysis is performed out of sample on two data sets, with and without the 2014 BTC return. The results of the survey are ambigous. The portfolio performance for the whole period goes in favor of including BTC in the portfolio. However, if the period of extremely high BTC returns is excluded from consideration, the portfolio has a lower risk-reward ratio than the portfolio without BTC.

One of the few studies where the results of previously considered studies are potentially refuted is conducted by Klein et al. (2018). To examine the cryptocurrency market's performance as a positive component in portfolio construction, the authors select traditional asset indices, and for each selected index, create and optimize one separate portfolio that includes BTC or gold. The goal of optimization is a portfolio with minimal variance. By comparing their performance, the authors conclude that BTC does not have the same investment characteristics like gold. A portfolio that includes BTC, e.g. The S&P 500-BTC, contains a small proportion of BTC, unlike the S&P

500-gold portfolio where there is a higher proportion of gold in the portfolio. The risk ratio, measured by standard deviation and CVaR, and the expected portfolio return, also prefers gold as an alternative asset for portfolio hedging.

All of the mentioned papers examine the reactions of incorporating one or more cryptocurrencies into a well-diversified portfolio composed of some form of traditional or alternative assets. However, given their number, the secondary cryptocurrency market can be viewed as a separate asset class and it is therefore desirable to examine the possibility of constructing an efficient portfolio made up solely of cryptocurrencies with different allocation goals. One of the first papers examining such a possibility is conducted by (Liu, 2018). The author creates six portfolios and pursues multiple optimization goals. The observed sample consists of ten cryptocurrencies with a market capitalization of more than 1 billion dollars over a four-year period, August 2015. - April 2018 period. The results are contrary to what was expected. Other than the portfolio with minimal variance, none of the optimization models met their target. The highest cumulative yield was achieved by the portfolio with the optimization goal of maximizing the utility function, and the highest Sharpe ratio was achieved by a portfolio with equally weighted assets, so the author concludes that in the cryptocurrency market sophisticated models cannot beat the performance of portfolios with equal weights - looking at them from the viewpoint of a rational investor.

Analysis of the options for optimization and diversification of risk in the cryptocurrency market is also conducted by (Brauneis and Mestel, 2018). The authors initially collect data from 500 and 20 most liquid cryptocurrencies, and create several different portfolios with associated optimization goals. The study is conducted out of sample with an initial set of observations of 183 historical daily returns used to estimate the parameters. Their results confirm the previous research. The highest expected return, as well as the Sharpe ratio, was achieved by the portfolio with equally weighted assets, regardless of the frequency of rebalancing. It concludes that an portfolio with equal allocations is the best choice when creating and modeling a portfolio in the cryptocurrency market.

The performance analysis of a sophisticated portfolio optimization model in relation to the passive approach of equal allocations in the cryptocurrency market is also conducted by Platanakis et al. (2019). Unlike previous papers, the analysis was conducted on weekly observations for only four cryptocurrencies: bitcoin, litecoin, ripple and dash for the period from February 2014 until January 2018 The optimization goal used was to maximize the utility function with the short sale limit, and Sharpe and omega ratios were used to evaluate performance. The study was conducted out of sample with two time periods for model training. Given the results of performance measures that do not favor either model, the authors conclude that passive (naive) diversification with equal allocations is a better choice for portfolio construction in the cryptocurrency market.

In all the research papers described above, the cryptocurrency market was viewed as one separate market in which cryptocurrencies have equal characteristics. Each cryptocurrency represented an input variable with equal probability of selection as a component of the portfolio, with its potential allocation in the portfolio defined by the optimization goal. In other words, the initial selection of portfolio components is either conditioned by an existing framework - such as the CRIX cryptocurrency index, or left to the choice resulting from the portfolio optimization of several different cryptocurrencies, most often cryptocurrencies with high market capitalization. Such an approach implies that all cryptocurrencies are equal in all their properties and capabilities, which is highly questionable. In this paper we test this common belief and approach the problem from another angle. Cryptocurrencies can be cathegorized in six basic categories: payment currencies, blockchain economies, utility tokens, privacy coins, stablecoins and others. Each category is specific and offers certain advantages over the other. Since utilization tokens provide a specific purpose in the practical application of a product or service, it is by far the most created on decentralized computer platforms, i.e. blockchain economies. Accordingly, the cryptocurrency market can also be viewed through sectoral division according to their utilization properties. Comparing portfolio performance with and without sectoral cryptocurrency selection will determine the usefulness of applying such an approach. In addition, cryptocurrencies that have a lower market capitalization and do not represent input variables in previous papers will be considered. In other words, cryptocurrencies that are undervalued by their fundamentals will be easier to spot by sectoral observation of the cryptocurrency market. Such an approach is necessary and desirable to eliminate the subordinated position of potential investors, that is, to contribute more to the performance of the investor portfolio in the cryptocurrency market. Also, with the aim of evaluating their performance, the performance of the resulting portfolios will be compared with the performance of the CRIX index over the same time period.

3. Data and Methodology

For the purpose of this study, we used publicly available daily price data (in USD) for a total of 65 cryptocurrencies collected from the Coinmarketcap - CMC platform pages, was used. Data was collected for the period from 8/26/2019 to 02/22/2020 creating a sample of a total of 146 daily observations, or 145 daily returns for 65 time series.

To test the utility of cryptocurrency sectoral division, an existing portfolio consisting of the top 50 cryptocurrencies by market capitalization includes additional 15 cryptocurrencies, 5 leading cryptocurrencies by each of the three leading utilization sectors by market capitalization: finance, exchanges and business services. Sectoral cryptocurrencies that entered the first 50 by market capitalization were excluded and replaced by the next utilization token by size of market capitalization in the respective sector.

We form multiple portfolios with different optimization goals of risk minimization, return maximization and maximization of return and risk ratios. Given the results of previous research by Briere et al. (2015) and Lee Kuo Chuen et al. (2018) and the absence of a normal distribution of returns, apart from the standard deviation, will use the conditional Value at Risk - CVaR for the risk measure, i.e. the methodology that follows the work of (Rockafellar and Uryasev, 2000), with a confidence level of 95%. Our optimization goals are as follows: minimum variance (MinVar), minimum CVaR (MinCVaR), maximize sharpe ratio (MaxSR), maximize stable tail-adjusted return ratio (MaxSTARR), maximize utility function (MaxUT) and maximize mean return (MaxMean). In order to examine the benefits of treating the cryptocurrency market through sector division we conduct the research in two steps. The first step is to form and test the performance of a portfolio whose components make up the first 50 cryptocurrencies by market capitalization. In the second step, an additional 15 sectoral cryptocurrencies are included in the existing data set. In order to achieve the inclusion of sector cryptocurrencies in the portfolio, in the second step, linear group constraints are created where 20% of the total portfolio allocation must be allocated to sector cryptocurrencies according to the optimization goals. The notation of portfolio optimization goals involving sector cryptocurrencies is as follows: minimum variance-sector (MinVar-S), minimum CVaR-sector (MinCVaR-S), maximize sharpe ratio-sector (MaxSR-S), maximize stable tail-adjusted return ratio- sector (MaxSTARR-S), maximize utility function-sector (MaxUT-S) and maximize mean return-sector (MaxMean-S). Optimization is performed out of sample (backtesting), with the same parameters for each optimization goal. A time period of $k = 10$ days was used to estimate the initial parameters and portfolio allocation. Given the dynamics of the cryptocurrency market, a more frequent monthly rebalance of $K = 30$ days was chosen with the so-called extending window approach $k + K$. For each period $k + 1$, portfolio returns are drawn with respect to the results of the allocation optimization in the previous k, i.e. $k + K$ moment.

3.1 Asset Allocation Models

The basic optimization model used in this paper is based on Modern Portfolio Theory, (Markowitz, 1952). In its original form, the model focuses on minimizing the variance of the asset portfolio for a given level of expected return within certain theoretical assumptions, which is why it is often referred to as the mean-variance (M-V) model. The basic form of the Markowitz formulation (soft return constraints) expressed in the form of linear algebra can be written as follows:

$$\min_{w} \quad \sigma_p^2\,(w) = w^{\mathrm{T}}\hat{\Sigma}w \qquad (1)$$
$$s.t. \quad \mathbf{1}_N^{\mathrm{T}}w = 1, \qquad x^T w \geq \mu, \qquad w_i \geq 0$$

where σ_p^2 is the variance of the portfolio, $w = (w_1, w_2, \dots, w_N)^T$ are the weights of individual assets in the portfolio and $\hat{\Sigma}$ is the estimated covariance matrix of assets N and their returns T. The above expression involves three additional constraints: $\mathbf{1}_N$ represents a $(Nx1)$ vector where all elements of the vector represent the portfolio weights and their sum must be one (full investment constraint), x is the $(Nx1)$ vector of the expected returns of the portfolio assets whose sum, with respect to individual portfolios of the portfolio assets, must be greater than or equal to the desired total portfolio return μ. The last restriction defines a constraint on short selling assets, that is, all portfolio holdings must be positive in size. By further formulation, the basic Markowitz model presented above is more adapted to the actual needs where its variants are used in this paper and described.

3.2 Global Minimum Variance Portfolio Objective

If the limit of the required rate of return is omitted from expression (1), portfolio optimization with the aim of minimizing risk results in a global minimum variance of portolio - GMV. Such a strategy is focused only the return covariance matrix and is based on finding the proportion of individual assets that minimizes the total variance of the portfolio. The GMV formulation used in this paper is given by expression (2), which includes a linear constraint for a sectoral cryptocurrency group. For portfolios that do not include sectors, the linear restriction is omitted.

$$\begin{gathered} \min_{w} \quad \sigma_p^2(w) = w^{\mathrm{T}}\hat{\Sigma}w \\ s.t. \quad \mathbf{1}_N^{\mathrm{T}}w = 1, \qquad w_i \geq 0, \qquad L \leq \mathrm{A}w \leq U \end{gathered} \tag{2}$$

where L and U are the lower and upper bounds for the sector cryptocurrency group. A is the constraint matrix for the sector cryptocurrency group.

3.3 Global Minimum CVaR Portfolio Objective

The disadvantage of the expression (2), is the assumption of a normal distribution of the portfolio's asset return for which the parameters are estimated. Considering the results of the study by (Briere et al., 2015) and (Lee Kuo Chuen et al., 2018), where evidence for the presence of a heavy-tailed cryptocurrency return distribution is shown, in this study expression (3) is used (Petukhina et al., 2018) and (Eisl, 2015), which is based on the CVaR methodology by (Rockafellar and Uryasev, 2000). We use a more reliable risk measure (CVaR) as a measure of risk so that the Mean-Variance model goes into Mean-Conditional Value at Risk (M-CVaR).
We define the cumulative distribution function of a loss function $z = f(w, y)$ as

$$\Psi(w, \zeta) = P\{y | f(w, y) \leq \zeta\} \tag{3}$$

Where w is fixed decision vector (i.e. portfolio weights), ζ loss associated with that vector and y uncertainties (e.g. market variables) that impact the loss. Then, for a given confidence level α, the Value at Risk (VaR_α) associated with portfolio is given as

$$VaR_{\alpha}(w) = min\{y|\Psi(w,\zeta) \geq \alpha\} \tag{4}$$

If $f(w, y)$ exceeds the VaR, then the expected value of the loss is defined as:

$$\mathrm{CVaR}_{\alpha}(w) = \frac{1}{1-\alpha} \int_{y(w) \leq VaR_{\alpha}(w)} yf(y|w)\, dy \tag{5}$$

Expression (5) is adapted to the optimization goal of risk minimization, with a confidence level of 95%. For portfolios that do not include sector cryptocurrencies, their linear restriction is also omitted.

$$\begin{gathered} \min_{w} \quad \mathrm{CVaR}_{\alpha}(w) \\ s.t. \quad \mathbf{1}_N^{\mathrm{T}} w = 1, \qquad w_i \geq 0, \qquad L \leq \mathrm{A}w \leq U \end{gathered} \tag{6}$$

3.4 Maximize Sharpe and STARR Ratio Portfolio Objective

The basic Markowitz relation (1) minimizes the variance of the portfolio return given the default expected return. On the other hand, by putting the expected return on the portfolio (adjusted for the risk-free interest rate over the same observation period) and the standard deviation of the portfolio, a Sharpe ratio will be obtained, i.e. a ratio indicating how much additional return is received per unit of risk. With the rational investment condition, by changing the investor's tolerance for risk, one will expect a higher expected return for the additional risk unit. In this case, the optimization goal of maximizing the return for a given level of risk is implemented. Portfolios that have the highest expected return for a given level of risk create an efficient frontier of feasible portfolios, hence the portfolio that has the highest Sharpe ratio represents the optimal portfolio, i.e. the tangent portfolio used in this paper (7). For portfolios without sector cryptocurrency, the linear restriction is omitted.

$$\begin{gathered} \max_{w} \quad \left\{\frac{w^{\mathrm{T}}\mu - \bar{r}_f}{\sqrt{w^{\mathrm{T}}\hat{\Sigma}w}}\right\} = \left\{\frac{w^{\mathrm{T}}\mu}{\sqrt{w^{\mathrm{T}}\hat{\Sigma}w}}\right\} \\ s.t. \quad \mathbf{1}_N^{\mathrm{T}} w = 1, \qquad w_i \geq 0, \qquad L \leq \mathrm{A}w \leq U \end{gathered} \tag{7}$$

Where $\bar{r}_f$ represents the risk-free interest rate adjusted for the observation period. For the purposes of this research, the risk-free interest rate is omitted, as can be seen from (7) and (8).

If CVaR is used in the denominator of expression (7) instead of standard deviation, the Sharpe ratio goes to Stable Tail-Adjusted Return Ratio (STARR) and is given by (8). The optimization goal is to maximize the STARR ratio with a 95% confidence level and a linear limit for sector cryptocurrencies.

$$\max_{w} \quad \left\{\frac{w^{\mathrm{T}}\mu-\bar{r}_f}{\mathrm{CVaR}_{\alpha}(w)}\right\}=\left\{\frac{w^{\mathrm{T}}\mu}{\mathrm{CVaR}_{\alpha}(w)}\right\} \tag{8}$$
$$s.t. \quad \mathbf{1}_N^{\mathrm{T}}w=1, \qquad w_i \geq 0, \qquad L \leq \mathrm{A}w \leq U$$

3.5 Maximize Quadratic Utility Function Portfolio Objective

The Markowitz model is also based on the assumption of a utility function, that is, the degree of satisfaction an investor achieves by investing in some form of asset. Specifically, the disadvantage of a Sharpe ratio is the assumption that all investors are equally risk-averse, resulting in only one optimal portfolio that delivers the best reward-risk ratio. On the other hand, by changing the ratio of expected return and risk, it is possible to create an indifference curve, i.e. an utility as a function of investor risk preference for lower but safer versus higher but riskier expected returns, whereas different combinations of risks and returns found on the indifference curve equal investor satisfaction. In order to derive the utility function curve, it is necessary to introduce an investor aversion parameter to risk γ. According to (9), a lower parameter value (lower risk aversion) also means a lower penalization of the portfolio risk contribution, which leads to a higher risk portfolio, that is, a potentially higher expected return. Conversely, in the case of more risk aversion, higher risk portfolios will be more penalized, leading to lower risk portfolios and lower expected returns. By gradually increasing the degree of risk aversion, a portfolio efficient frontier is derived in order to find the desired risk-return profile. The value of the parameter used in the paper is one.

$$\max_{w} \quad \mu(w)-\frac{\gamma}{2}w\hat{\Sigma}w \tag{9}$$
$$s.t. \quad \mathbf{1}_N^{\mathrm{T}}w=1, \qquad w_i \geq 0, \qquad L \leq \mathrm{A}w \leq U$$

where γ represents the degree of investor aversion to risk.

3.6 Maximize Return Portfolio Objective

In contrast to the strategy that minimizes variance, the study also implements an optimization strategy that maximizes expected portfolio returns, i.e. does not include a predefined level of risk. In this case, the optimization algorithm does not take into account the variance and covariance matrix, but uses the average returns of the previous period to estimate the highest expected portfolio return in the next period. The assets of the portfolio with the highest average return in the previous period will have the highest allocation in the portfolio. Because the strategy does not consider risk as an input in optimization, it is considered extremely high-risk. The formulation used in this paper to maximize the expected return is given by (10). It includes a linear constraint for a group of sector cryptocurrencies. For portfolios without cryptocurrencies divided by sectors, the linear restriction is not included.

$$\max_{w} \quad \mu_p(w) = w^{\mathrm{T}} \qquad (10)$$
$$s.t. \quad \mathbf{1}_N^{\mathrm{T}} w = 1, \qquad w_i \geq 0, \qquad L \leq \mathrm{A}w \leq U$$

Where je μ_p is the expected portfolio return.

3.7 Performance Metrics

The results of several different absolute and relative measures of success are presented in order to evaluate the success of each optimization strategy: Sharpe ratio, MSquared, Regression alpha, Jensen's alpha, Treynor ratio and Information ratio, where the values are calculated annually and relate to total time series of portfolio returns. The annual average geometric return was used to calculate the realized portfolio return: $R_{Gi} = prod(1 + R_{di})^{\frac{scale}{n}} - 1$, where R_{di} is the daily realized return of the observed portfolio i at time t, n is the total number of existing observations and $scale$ number of observations in a year 252. The annual standard deviation is given by the $\sigma_{ai} = \sigma_{di} \times \sqrt{252}$ where σ_{di} is the standard deviation of daily portfolio returns. If the risk-free interest rate is omitted (currently it stands at a historically low level so this is not problematic even from the theoretical viewpoint) from the calculation as an indicator of opportunity profitability, the Sharpe ratio SR used in this paper is given by (11).

$$SR = \frac{R_{Gi}}{\sigma_{ai}} \qquad (11)$$

Apart from ranking investment opportunities, the Sharpe ratio does not provide additional information. Only for investments with the same level of risk, the Sharpe ratio indicates exactly how much one investment is better than the other. The measure that corrects this shortcoming is Msquared M^2 (12). It indicates the difference in the ratio of return to risk between investments, regardless of the level of risk. Risk-free interest rate is omitted.

$$M^2 = R_{Gi} \times \frac{\sigma_{aP}}{\sigma_{ai}} \qquad (12)$$

Where je σ_{aP} is the annual standard deviation of the market (in this case of the CRIX index). By comparing the values of the above measures, information can be obtained about the superiority of a specific strategy over others in the cryptocurrency market. However, the above measures were calculated in the sample, i.e. on the total sample of data. The results show the average values of the measures calculated on the portfolio returns of each optimization strategy. In order to consider investment opportunities, it is also advisable to consider the results of the linear regression model (13) between the time series of portfolio returns as dependent variables and the CRIX index return as an independent variable. Where CRIX index return represents the benchmark of the market, since the estimated slope value is an input parameter for the Treynor ratio (14) and Jensen's alpha (15).

$$R_{di} = a_i + \beta_i \times R_{dM} + \epsilon_i \tag{13}$$

Where a_i is the regression intercept, β_i is the slope of the regression line, R_{dM} is the daily realized market return (CRIX index) and ϵ_i is the residual deviation from the regression line. Substituting the standard deviation in expression (10) for the estimated slope value of the regression equation $\bar{\beta}_i$ as a measure of volatility risk, we obtain the Treynor ratio TR. The measure used in this paper is given by (14).

$$TR = \frac{R_{Gi}}{\bar{\beta}_i} \tag{14}$$

As part of the Capital Asset Pricing Model CAPM (Sharpe, 1963), the slope value of (13) is the estimated magnitude of the regression equation that relates the risk of a potential investment to the equilibrium expected return on a risky investment. If the potential investment is more variable relative to the market, rational investors will expect a premium on their investment to compensate for the higher risk assumed and the estimated slope of the direction $\bar{\beta}_i$ will be above one. The basic relation of the CAPM model indicates that the expected return rate on investment equals the risk-free interest rate increased by the market risk premium (benchmark standard), which is corrected by its systematic risk, i.e. $\bar{\beta}_i$. In addition, the CAPM model neglects the existence of a specific investment risk ϵ_i, that is, it only takes into account its systematic risk arising from the movement of the entire market. By rearranging the basic equation of the CAPM model, one can derive expression (15) representing Jensen's alpha. Jensen's alpha indicates whether an investment has achieved a higher or lower return than the expected or required return per CAPM model. Assuming that CRIX index is an adequate approximation of the cryptocurrency market movement, if we obtain portfolios with positive Jensen's alpha the optimization strategy can be viewed as having outperformed the market by yielding higher returns than required by the CAPM model. In (15), the risk-free interest rate is also omitted.

$$\alpha_i = R_{Gi} - \beta_i \times R_{GM} \tag{15}$$

Where je R_{GM} is the annual average geometric return of the CRIX index. The last measure of performance used in this study is the Information ratio IR (16). Information ratio is a relative measure that compares the difference between the annual average geometric portfolio return and the CRIX index (active premium) and the annual standard deviation of the active premium between the portfolio yield and the CRIX index (tracking error). If IR has a positive value, it would mean that the optimization strategy performed better than the CRIX index, with CRIX index being a good proxy for the general movement of the cryptocurrency market.

$$IR = \frac{R_{Gi} - R_{GM}}{\sigma_{iM}} \tag{16}$$

Where je σ_{iM} is the standard deviation of the active premium between the portfolio return and the CRIX index. In addition, the study presents the cumulative portfolio return of individual optimization strategies and CRIX index assuming an initial investment of one dollar, as well as their worst drawdown, which indicates the highest loss relative to the highest value of cumulative return in the observed period.

4. Results

We present and interpret the obtained empirical results in two phases. In the first phase, the results are reviewed and interpreted by a comparative method between asset allocation models according to the initial selection of the portfolio components. In addition, the success of a particular strategy is judged by the implementation of performance measures that include the CRIX index as a benchmark for the crypto market over the observation period. In second phase, the results of allocation models are compared and interpreted between portfolios to determine the benefits of dividing and optimizing cryptocurrencies according to their appropriate sectors.

Table 1 shows the results of the previously described performance measures for six asset allocation models for 50 cryptocurrencies selected by CMC market size. Table 2 shows the results of portfolio performance measures that, in addition to the 50 cryptocurrencies per CMC, include an additional 15 cryptocurrencies per related financial, exchange and business services sector. The last column in the tables shows the results of the CRIX index over the same period as the benchmark of the cryptocurrency market. All values except the regression beta and worst drawdown of each optimization strategy are reported annually. In case of negative values the Traynor ratio is omitted.

If the regression beta between portfolio returns and the CRIX index is considered in the context of the CAPM model, most portfolios have a very low systematic risk. Moreover, four strategies have achieved a negative beta, which means that they are moving in the opposite direction of the CRIX index. Positive regression alphas indicate that, in the case of a cryptocurrency market stagnation, each of the observed portfolios on average achieves higher returns than the market returns. As expected, the highest average alpha was achieved by the MaxMean portfolio. On the other hand, it is worth pointing out that the highest annual geometric return as well as the cumulative return in the total period, was achieved by portfolio with the optimization goal of minimizing CVaR. Therefore we conclude that the best values of the performance measures are expected to be related to the MinCVaR portfolio. So SR stands at a high of 2,68 which is by far the best of all other optimization strategies. The difference between the MSquared return and the risk ratio relative to the CRIX index also benefits the portfolio, which minimizes the conditional value at risk. Jensen's alpha which suggests if the strategy has outperformed the market for all optimization solutions is positive. In other words, all strategies yielded returns higher than required by the CAPM model. The highest ratio of active premium IR and standard deviation was also achieved by the MinCVaR portfolio.

Table 1. Asset allocation models without sectors cryptocurrencies

		Asset Allocation Models						
Performance Metrics		MinVar	MinCVaR	MaxSR	MaxSTARR	MaxUT	MaxMean	CRIX
Beta	β_i	0,05	-0,05	0,02	-0,014	-0.05	-0,01	1
Annualized Alpha	a_{ai}	1,12	1,91	1,16	1,53	0,97	2,29	/
Annualized Return	R_{Gi}	0,94	1,44	0,95	0,62	0,72	0,95	0,57
Annualized Std Dev	σ_{ai}	0,49	0,54	0,48	0,94	0,46	1,04	0,47
Worst Drawdown	WD	0,27	0,29	0,26	0,57	0,27	0,56	0,31
Cumulative Return	CY	1,46	1,67	1,47	1,32	1,37	1,47	1,29
Sharpe Ratio	SR	1,92	2,68	1,95	0,66	1,58	0,91	1,20
MSquared	M^2	0,91	1,27	0,92	0,31	0,75	0,43	0,57
Treynor Ratio	TR	18,69	/	59,03	/	/	/	0,57
Jensen's Alpha	α_i	0,91	1,47	0,94	0,63	0,75	0,95	/
Information Ratio	IR	0,56	1,20	0,56	0,05	0,23	0,34	/

On the other hand, the lowest geometric and cumulative returns were achieved by an optimization strategy for maximizing the ratio of returns and CVaR, so the values of other performance measures for MaxSTARR strategy are also lower than performance measure of other strategies. The highest annual standard deviation was achieved by the portfolio with the optimization goal of maximizing expected returns, and the lowest portfolio maximizing the utility function. In comparison with the CRIX index, all implemented optimization goals achieved a higher cumulative return in the same observation period. However, the CRIX index achieved a lower standard deviation level in five out of six observed cases. Four portfolios have smaller worst drawdowns than the index as well as a higher SR.

Figure 1 shows the dynamics of the daily cumulative returns of individual strategies, the total daily returns of all strategies, and an underwater chart for drawdown to further illustrate the performance of portfolio optimization goals.

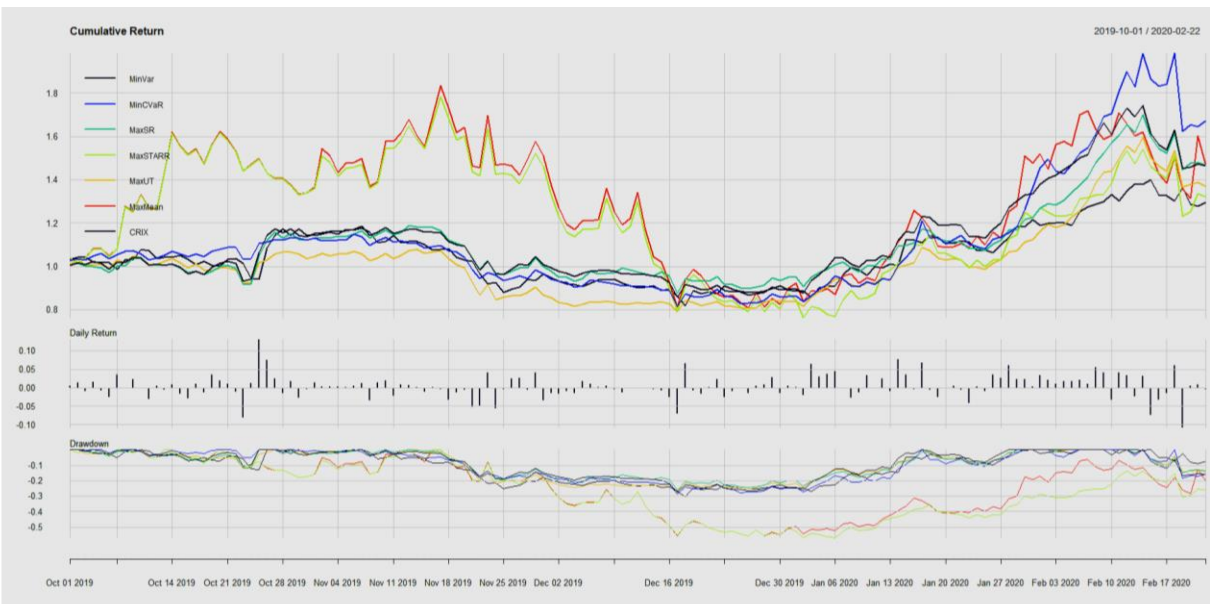

Fig. 1. Performance summary various strategies without sectors cryptocurrencies

By including an additional 15 sectoral cryptocurrencies that would not initially be selected as a component of the portfolios by their market capitalization, the results differ by all measures shown in Table 2. Similar to the above, the regression beta values suggest low systematic risk of all optimization strategies. The highest average annual alpha, geometrical as well as the total cumulative return, was achieved by the portfolio with the aim of maximizing it. Which is in line with expectations due to the higher risk assumed as standard deviation, i.e. worst drawdown. However, the return of the MaxMean-S portfolio adequately compensated for the higher risk assumed, which ultimately resulted in a high SR of 4,66. MSquared also points out the difference between the MaxMean-S portfolio and the CRIX index. Jensen's alpha suggests that all observed portfolios have yielded higher than expected returns per CAPM ratio. The significant difference between the realized geometric return of the MexMean-S portfolio and the CRIX index, comparing to the standard deviation of the active premium which is extremely low due to the equal volatility between the observed investments, also influenced the highly positive Information ratio. In the second order of the best size of all performances, except for the risk measures, it was achieved by a portfolio that maximizes the ratio of return and risk expressed as CVaR.

Table 2. Asset allocation models with sectors cryptocurrencies

		Asset Allocation Models						
Performance Metrics		MinVar -S	MinCVaR-S	MaxSR -S	MaxSTAR R-S	MaxUT -S	MaxMean-S	CRIX
Beta	β_i	0,05	0,03	-0,08	0,10	-0,12	0,22	1
Annualized Alpha	a_{ai}	0,86	2,39	1,37	3,74	1,51	10,10	/
Annualized Return	R_{Gi}	0,73	1,99	1,09	3,02	1,09	5,84	0,57
Annualized Std Dev	σ_{ai}	0,45	0,53	0,43	0,67	0,48	1,17	0,47
Worst Drawdown	WD	0,27	0,22	0,29	0,23	0,25	0,35	0,31
Cumulative Return	CY	1,37	1,88	1,52	2,23	1,53	3,02	1,29
Sharpe Ratio	SR	1,62	3,79	2,53	4,50	2,27	4,99	1,20
MSquared	M^2	0,77	1,79	1,20	2,13	1,07	2,36	0,57
Treynor Ratio	TR	14,91	76,54	/	31,22	/	26,88	0,57
Jensen's Alpha	α_i	0,70	1,98	1,12	2,97	1,16	5,72	/
Information Ratio	IR	0,26	2,04	0,77	3,09	0,74	4,31	/

In terms of performance measures, the strategy to minimize standard deviation of the portfolio has performed the worst. On the other hand, the lowest standard deviation was achieved by the MaxSR-S optimization strategy, where the standard deviation of the MinVar-S portfolio is slightly higher. The lowest worst drawdown belongs to the MinCVaR-S portfolio, which is in line with the optimization goal. In comparison with the CRIX index, all of the optimization goals achieved a higher cumulative return in the same observation period. It is also worth pointing out that only two

MaxUT-S and MaxMean-S strategies achieved a higher standard deviation than the CRIX index. In addition, all portfolios achieved a higher Sharpe ratio than the CRIX index during the same period.

Figure 2 shows the dynamics of the daily cumulative returns of individual strategies, the total daily returns of all strategies, and the underwater chart for drawdown, further illustrating the performance of portfolio optimization goals.

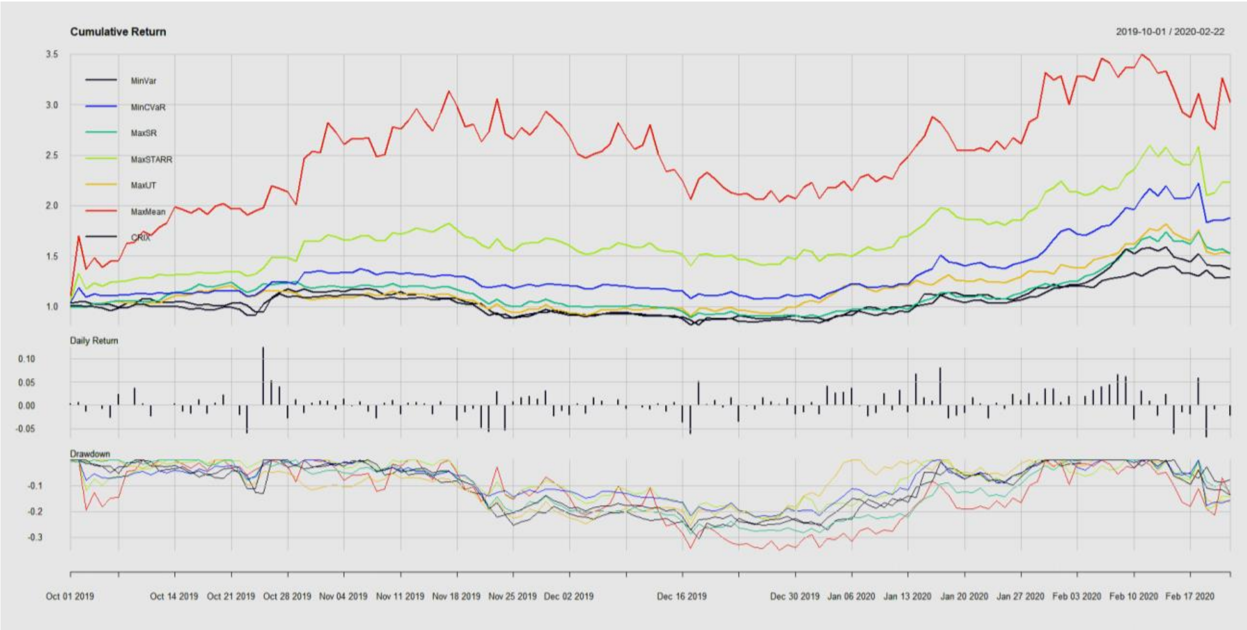

Fig. 2. Performance summary various strategies with sectors cryptocurrencies

5. Interpretation of the results and discussion

Interpreting the results involves considering the performance of strategies between portfolios that differ in composition. The first thing to notice is the height of the regression alpha, which for all portfolios except the MinVar-S, achieved a better result if sectoral cryptocurrencies were included in the portfolio. Portfolios with additional cryptocurrencies earn on average more returns than portfolios without sectoral components. Geometric return has the same relationships. Only the MinVar-S portfolio has a lower return than the portfolio return without sectoral cryptocurrency, thus confirming our finding and logic that there are benefits in treating the cryptocurrency market through sectoral affiliation.

In terms of risk, four strategies involving sectoral cryptocurrencies have achieved a lower standard deviation than portfolios without them. However, the higher risk was offset by the higher return achieved, implying a higher Sharpe ratio. Worst drawdown also points to the benefits of including additional cryptocurrencies where only the MaxSR-S portfolio has a higher worst drawdown than the MaxSR portfolio.

The inclusion of sector cryptocurrencies has also led to an increase in cumulative return for all strategies except the MinVar-S portfolio. The biggest difference was recorded by the MaxMean-S portfolio, where its cumulative return increased by 1,55. By applying a return maximization strategy and considering sectoral cryptocurrencies as a components of the portfolio, it was possible to achieve a cumulative return higher by 105% over a 146-day period than the same strategy that does not consider sectoral cryptocurrencies. A significant increase in cumulative return was also achieved by the MaxSTARR-S portfolio of 0,91, or 69%, compared to MaxSTARR. The results of the Sharpe ratio and MSquared measures are consistent with the above findings. The largest increase in SR and M^2 refers to the MaxMean-S portfolio. Only the MinVar-S portfolio has a lower SR and M^2 relative to an equivalent strategy with no additional cryptocurrencies. Given that five of the six portfolios that include sectoral cryptocurrencies had a higher geometric return and the regression beta did not increase significantly, so Jensen's alpha performed better for all portfolios except MinVar-S. The inclusion of additional sector cryptocurrencies in existing portfolios contributes to the improvement of portfolio performance compared to the market represented by the CRIX index. Treynor ratio and Information ratio also performed significantly better for all sector portfolios except MinVar-S portfolios. Considering all the above, it can be concluded that five of the six portfolios created according to different optimization goals achieved better results if they view the cryptocurrencies through the sectoral perspective (financial, exchange and business services). Such results contribute significantly to the research of investment opportunities in the cryptocurrency market and sectoral segmentation of cryptocurrency market. In addition, the positive results point to two more observations from which certain conclusions need to be drawn.

The first observation is certainly the existence of distinction within the category of cryptocurrencies and also the category of utilization tokens. If cryptocurrency litecoin is used solely as a means of payment, comparing litecoin with a decentralized computer platform like ethereum and treating the two assets equally in the context of investment opportunities, is simply not practical or even impossible and this also confirmed by our results. On blockchain economies like ethereum, among other things, tokens as a type of cryptocurrency, can be created to provide a different purpose and utility in the practical application of the product and service. This also means that products and services are different, i.e. utilization tokens differ in their fundamentals. For an example, a utilization token with a strictly defined purpose in a specific online game should not be viewed as equal with a utilization token that has a wider purpose, such as tokens created for decentralized finances. If viewed together, the market recognizes such anomalies and "properly" values the assets observed. This situation has led to significantly better portfolio results when including sectoral cryptocurrencies.

In the cryptocurrency market beginning of 2020 and in 2019, was marked by a rise in the value of utilization tokens, which to some extent represented Decentralized Finance - DeFi. None of the 15 additional cryptocurrencies (tokens) selected by sector were in the top 50 by CMC, so they have not been considered in previous studies. Previous research papers have considered cryptocurrencies as a homogenous asset and relied solely on the general market optimization algorithm when selecting portfolio components. Such an approach implies a consensus on the magnitude of the equilibrium expected return of the selected cryptocurrencies. However, previously it has been shown that such a return does not even exist due to the absence of adequate valuation, that is, the intrinsic value of cryptocurrencies. Taking into account previous research, if one draws a parallel with thinking of the traditional capital market and the CAPM model, it can be concluded that all cryptocurrencies are properly valued, i.e. all cryptocurrencies are on the Security Market Line-SML. Our results in this paper suggest the opposite. Our conclusions are supported by positive average regression and realized alpha portfolios, as well as portfolios with additional sectoral cryptocurrencies.

In line with the obtained results, our findings emphasize the utility and necessity of observing the cryptocurrency market by sectoral affiliation with the aim of finding potentially “undervalued” cryptocurrencies. If portfolio components are selected solely by market capitalization, it would mean that these cryptocurrencies have already achieved the value that makes them a potential portfolio component. The possibility of price growth of such cryptocurrency is certainly much lower than the possibility of cryptocurrency growth which ranks much lower in terms of market capitalization. Sectorally, cryptocurrencies with much lower market capitalization are emerging and investors can more easily spot them. Looking at the overall capitalization of the sector, it is easier to spot and identify current trends in the cryptocurrency market, as was the growth trend in 2019 of DeFi cryptocurrencies.

6. Conclusion

Examining the utility of observing cryptocurrencies through their sectoral affiliation when constructing a portfolio is the primary theme of this paper. The results of the methodological approach are contributing to considering investment opportunities in the cryptocurrency market. The methodology for exploring the benefits of sectoral allocation and portfolio construction has been implemented in two phases. In the first phase, the performance of the portfolio limited in composition to market capitalization is created and interpreted. In the second phase the cryptocurrencies of the three leading sectors by market capitalization are included: finance, exchanges and business services. Consideration of the cryptocurrency market by sectoral affiliation is justified by the theoretical assumption that there are significant price trends of certain sectors in the cryptocurrency market. Such an approach easily recognizes cryptocurrencies that belong to the same sector and have lower market capitalization (higher price growth potential).

The results suggest that portfolios in which 20% of the weight is allocated to cryptocurrencies of lower market capitalization achieve higher values across all implemented performance measures in five of the six optimization strategies. It can be concluded that it is desirable and necessary to observe the cryptocurrency market through their type or their utility, and such an approach can be achieved by categorizing cryptocurrencies into their sectors. Potential investors, and portfolio managers in particular, should not consider cryptocurrencies only based on their market capitalization. Cryptocurrencies have characteristics and capabilities that define them according to their nominal purpose. Accordingly, portfolio managers are encouraged to consider cryptocurrencies by their characteristics (the type and purpose they provide) when constructing a portfolio, in order to eliminate their subordinate position and to contribute to portfolio performance in the cryptocurrency market.

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